chapter five

INDUCTION (ASYNCHRONOUS) MACHINES

The induction machine is the most rugged and the most widely used machine in industry. Like the dc machine discussed in the preceding chapter, the induction machine has a stator and a rotor mounted on bearings and separated from the stator by an air gap. However, in the induction machine both stator winding and rotor winding carry alternating currents. The alternating current (ac) is supplied to the stator winding directly and to the rotor winding by induction—hence the name induction machine.

The induction machine can operate both as a motor and as a generator. However, it is seldom used as a generator supplying electrical power to a load. The performance characteristics as a generator are not satisfactory for most applications. The induction machine is extensively used as a motor in many applications.

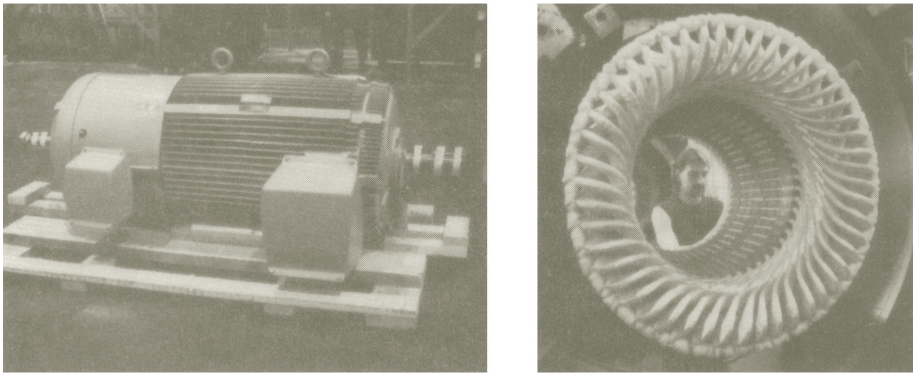
The induction motor is used in various sizes. Small single-phase induction motors (in fractional horsepower rating; see Chapter 7) are used in many household appliances, such as blenders, lawn mowers, juice mixers, washing machines, refrigerators, and stereo turntables. Large three-phase induction motors (in tens or hundreds of horsepower) are used in pumps, fans, compressors, paper mills, textile mills, and so forth.

The linear version of the induction machine has been developed primarily for use in transportation systems.

The induction machine is undoubtedly a very useful electrical machine. Single-phase induction motors are discussed in Chapter 7. Two-phase induction motors are used primarily as servomotors in a control system. These motors are discussed in Chapter 8. Three-phase induction motors are the most important ones, and are most widely used in industry. In this chapter, the operation, characteristic features, and steady-state performance of the three-phase induction machine are studied in detail.

5.1 CONSTRUCTIONAL FEATURES

Unlike dc machines, induction machines have a uniform air gap. A pictorial view of the three-phase induction machine is shown in Fig. 5.1*a*. The stator is composed of laminations of high-grade sheet steel. A three-phase winding is put in slots cut on the inner surface of the stator frame as shown in Fig. 5.1*b*. The rotor also consists of laminated ferromagnetic material, with slots cut on the outer surface. The rotor winding may be either of two types, the *squirrel-cage type* or the *wound-rotor type*. The squirrel-cage winding consists of aluminum or copper bars embedded in the rotor slots and shorted at both ends by aluminum or copper end rings as shown in Fig. 5.2*a*. The wound-rotor winding has the same form as the stator winding. The terminals of the rotor winding are connected to three slip rings, as shown in Fig. 5.2*b*. Using stationary



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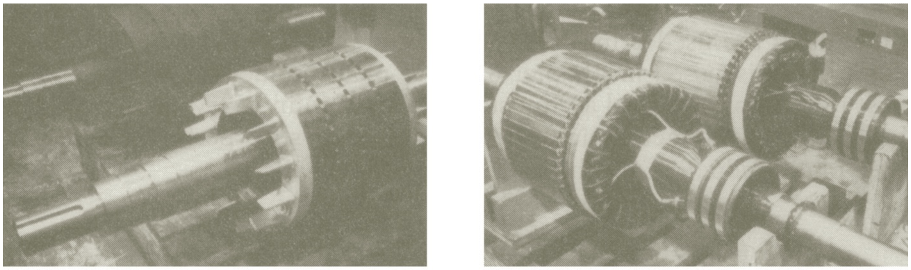
(*a*) (*b*)

FIGURE 5.1 Three-phase induction machine. (*a*) Induction machine with enclosure. (*b*) Stator with three-phase winding.



Courtesy of Westinghouse Canada Inc.

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(*a*) (*b*)

FIGURE 5.2 Rotor of an induction machine. (*a*) Squirrel-cage rotor. (*b*) Wound-rotor type.

brushes pressing against the slip rings, the rotor terminals can be connected to an external circuit. In fact, an external three-phase resistor can thus be connected for the purpose of speed control of the induction motor, as shown in Fig. 5.39 and discussed in Section 5.13.6. It is obvious that the squirrel-cage induction machine is simpler, more economical, and more rugged than the wound-rotor induction machine.

The three-phase winding on the stator and on the rotor (in the wound-rotor type) is a distributed winding. Such windings make better use of iron and copper and also improve the mmf waveform and smooth out the torque developed by the machine. The winding of each phase is distributed over several slots. When current flows through a distributed winding, it produces an essentially sinusoidal space distribution of mmf. The properties of a distributed winding are discussed in Appendix A.

Figure 5.3*a* shows a cross-sectional view of a three-phase squirrel-cage induction machine. The three-phase stator winding, which in practice would be a distributed winding, is represented by three concentrated coils for simplicity. The axes of these coils are 120 electrical degrees apart. Coil aa′ represents all the distributed coils assigned to the phase-a winding for one pair of poles. Similarly, coil bb′ represents the phase-b distributed winding, and coil cc′

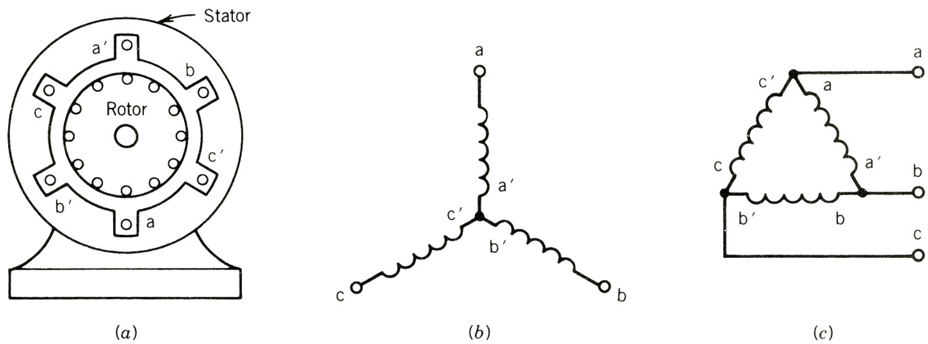


FIGURE 5.3 Three-phase squirrel-cage induction machine. (*a*) Cross-sectional view. (*b*) Y-connected stator winding. (*c*) Δ-connected stator winding.

represents the phase-c distributed winding. The ends of these phase windings can be connected in a wye (Fig. 5.3*b*) or a delta (Fig. 5.3*c*) to form the three-phase connection. As shown in the next section, if balanced three-phase currents flow through these three-phase distributed windings, a rotating magnetic field of constant amplitude and speed will be produced in the air gap and will induce current in the rotor circuit to produce torque.

5.2 ROTATING MAGNETIC FIELD

In this section we study the magnetic field produced by currents flowing in the polyphase windings of an ac machine. In Fig. 5.4*a* the three-phase windings, represented by aa′, bb′, and cc′, are displaced from each other by 120 electrical degrees in space around the inner circumference of the stator. A two-pole machine is considered. The concentrated coils represent the actual distributed windings. When a current flows through a phase coil, it produces a sinusoidally distributed mmf wave centered on the axis of the coil representing the phase winding. If an alternating current flows through the coil, it produces a pulsating mmf wave, whose amplitude and direction depend on the instantaneous value of the current flowing through the winding. Figure 5.4*b* illustrates the mmf distribution in space at various instants due to an alternating current flow in coil aa′. Each phase winding will produce similar sinusoidally distributed mmf waves, but displaced by 120 electrical degrees in space from each other.

Let us now consider a balanced three-phase current flowing through the three-phase windings. The currents are

$$i_{\mathrm{a}} = I_{\mathrm{m}} \cos \omega t \tag{5.1}$$

$$i_{\mathrm{b}} = I_{\mathrm{m}} \cos(\omega t - 120^\circ) \tag{5.2}$$

$$i_{\mathrm{c}} = I_{\mathrm{m}} \cos(\omega t + 120^\circ) \tag{5.3}$$

These instantaneous currents are shown in Fig. 5.4*c*. The reference directions, when positive-phase currents flow through the windings, are shown by dots and crosses in the coil sides in Fig. 5.4*a*. When these currents flow through the respective phase windings, each produces a sinusoidally distributed mmf wave in space, pulsating along its axis and having a peak located

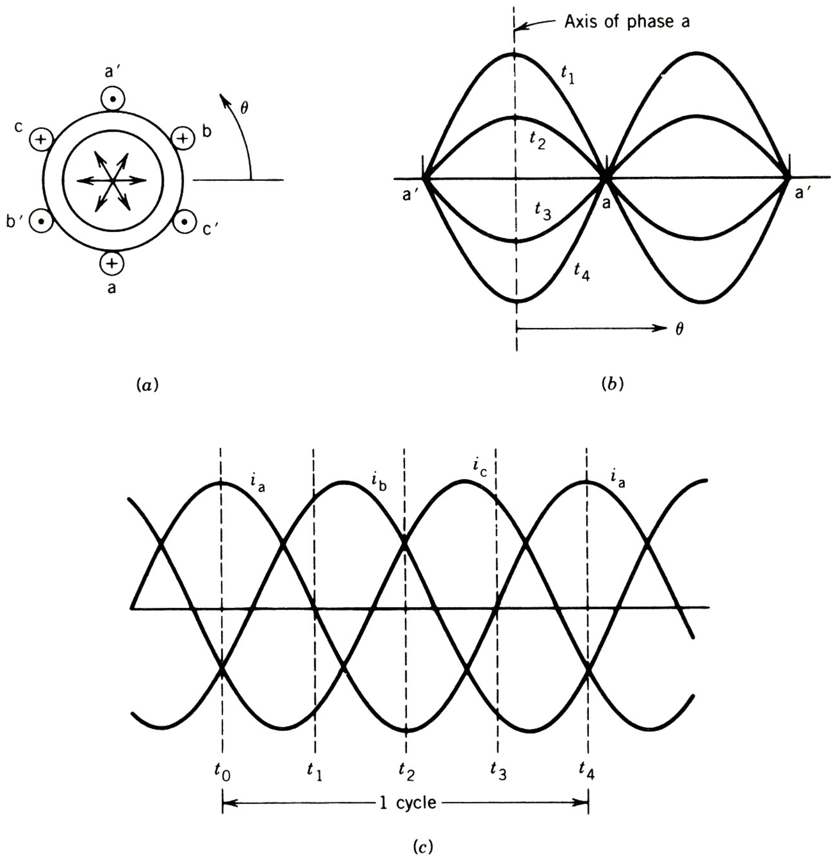


FIGURE 5.4 Pulsating mmf.

along the axis. Each mmf wave can be represented by a space vector along the axis of its phase with magnitude proportional to the instantaneous value of the current. The resultant mmf wave is the net effect of the three component mmf waves, which can be computed either graphically or analytically.

5.2.1 GRAPHICAL METHOD

Let us consider situations at several instants of time and find out the magnitude and direction of the resultant mmf wave. From Fig. 5.4*c*, at instant $t = t_0$, the currents in the phase windings are as follows:

$$i_{\mathrm{a}} = I_{\mathrm{m}} \quad \text{flowing in phase winding a} \tag{5.4}$$

$$i_{\mathrm{b}} = -\frac{I_{\mathrm{m}}}{2} \quad \text{flowing in phase winding b} \tag{5.5}$$

$$i_{\mathrm{c}} = -\frac{I_{\mathrm{m}}}{2} \quad \text{flowing in phase winding c} \tag{5.6}$$

The current directions in the representative coils are shown in Fig. 5.5*a* by dots and crosses. Because the current in the phase-a winding is at its maximum, its mmf has its maximum value and is represented by a vector $F_a = F_{max}$ along the axis of phase a, as shown in Fig. 5.5*a*. The mmfs of phases b and c are shown by vectors F_a and F_c, respectively, each having magnitude $F_{max}/2$ and shown in the negative direction along their respective axes. The resultant of the three vectors is a vector $F = \frac{3}{2}F_{max}$ acting in the positive direction along the phase-a axis. Therefore, at this instant, the resultant mmf wave is a sinusoidally distributed wave, which is the same as that due to phase-a mmf alone, but with $\frac{3}{2}$ the amplitude of the phase-a mmf wave. The component mmf waves and the resultant mmf wave at this instant ($t = t_0$) are shown in Fig. 5.5*b*.

At a later instant of time t_1 (Fig. 5.4*c*), the currents and mmfs are as follows:

$$i_a = 0, \qquad F_a = 0 \tag{5.7}$$

$$i_b = \frac{\sqrt{3}}{2} I_m, \qquad F_b = \frac{\sqrt{3}}{2} F_{max} \tag{5.8}$$

$$i_c = \frac{\sqrt{3}}{2} I_m, \qquad F_c = \frac{\sqrt{3}}{2} F_{max} \tag{5.9}$$

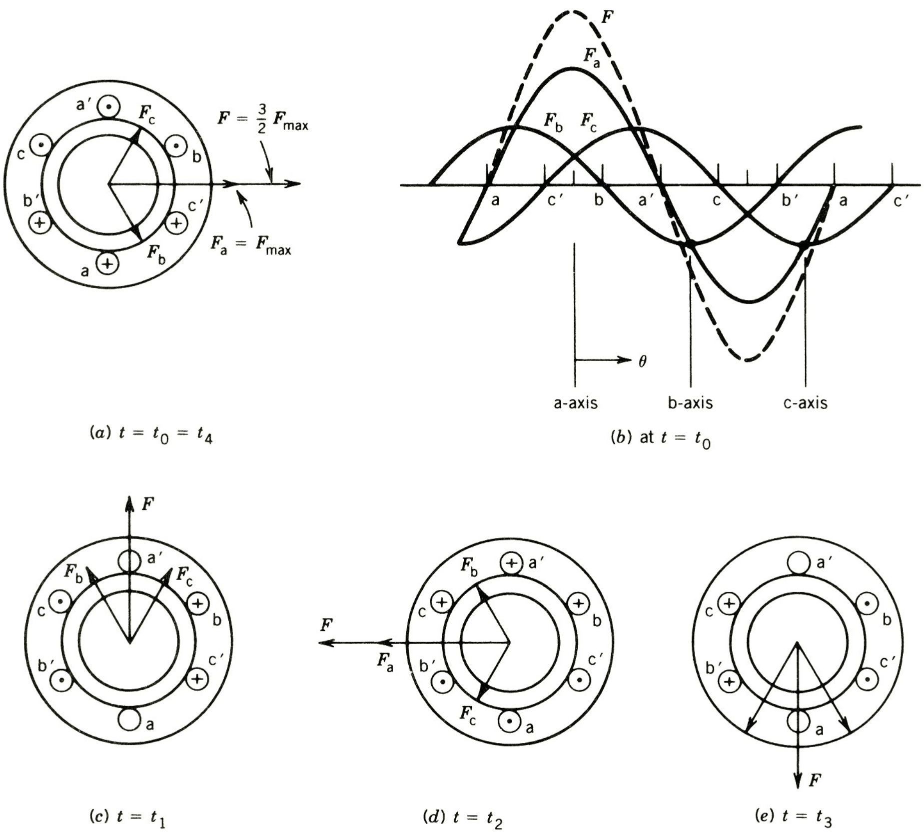


FIGURE 5.5 Rotating magnetic field by graphical method: mmfs at various instants.

The current directions, component mmf vectors, and resultant mmf vector are shown in Fig. 5.5*c*. Note that the resultant mmf vector F has the same amplitude $\frac{3}{2}F_{max}$ at $t = t_1$ as it had at $t = t_0$, but it has rotated counterclockwise by 90° (electrical degrees) in space.

Currents at other instants $t = t_2$ and $t = t_3$ are also considered, and their effects on the resultant mmf vector are shown in Figs. 5.5*d* and 5.5*e*, respectively. It is obvious that as time passes, the resultant mmf wave retains its sinusoidal distribution in space with constant amplitude, but moves around the air gap. In one cycle of the current variation, the resultant mmf wave comes back to the position of Fig. 5.5*a*. Therefore, the resultant mmf wave makes one revolution per cycle of the current variation in a two-pole machine. In a p-pole machine, one cycle of variation of the current will make the mmf wave rotate by $2/p$ revolutions. The revolutions per minute n (rpm) of the traveling wave in a p-pole machine for a frequency f cycles per second for the currents are

$$n = \frac{2}{p} f 60 = \frac{120f}{p} \tag{5.10}$$

It can be shown that if i_a flows through the phase-a winding but i_b flows through the phase-c winding and i_c flows through the phase-b winding, the traveling mmf wave will rotate in the clockwise direction. Thus, a reversal of the phase sequence of the currents in the windings makes the rotating mmf rotate in the opposite direction.

5.2.2 ANALYTICAL METHOD

Again we shall consider a two-pole machine with three phase windings on the stator. An analytical expression will be obtained for the resultant mmf wave at any point in the air gap, defined by an angle θ. The origin of the angle θ can be chosen to be the axis of phase a, as shown in Fig. 5.6*a*. At any instant of time, all three phases contribute to the air gap mmf along the path defined by θ. The mmf along θ is

$$F(\theta) = F_a(\theta) + F_b(\theta) + F_c(\theta) \tag{5.11}$$

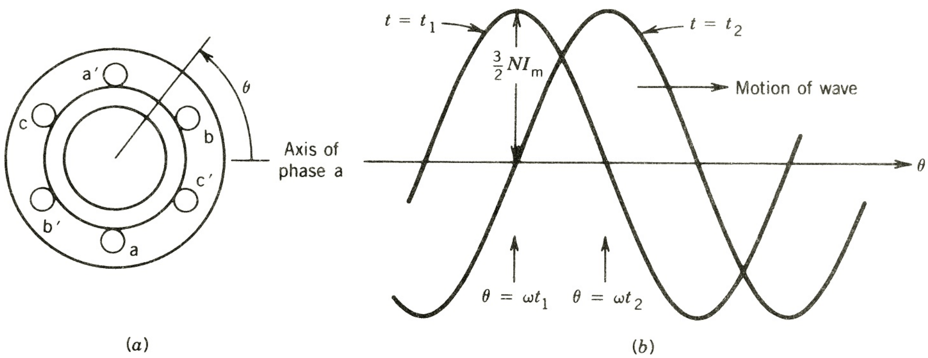


FIGURE 5.6 Motion of the resultant mmf.

At any instant of time, each phase winding produces a sinusoidally distributed mmf wave with its peak along the axis of the phase winding and amplitude proportional to the instantaneous value of the phase current. The contribution from phase a along θ is

$$F_{\mathrm{a}}(\theta) = Ni_{\mathrm{a}} \cos\theta \tag{5.12}$$

where N is the effective number of turns in phase a
i_{a} is the current in phase a

Because the phase axes are shifted from each other by 120 electrical degrees, the contributions from phases b and c are, respectively,

$$F_{\mathrm{b}}(\theta) = Ni_{\mathrm{b}} \cos(\theta - 120°) \tag{5.13}$$

$$F_{\mathrm{c}}(\theta) = Ni_{\mathrm{c}} \cos(\theta + 120°) \tag{5.14}$$

The resultant mmf at point θ is

$$F(\theta) = Ni_{\mathrm{a}} \cos\theta + Ni_{\mathrm{b}} \cos(\theta - 120°) + Ni_{\mathrm{c}} \cos(\theta + 120°) \tag{5.15}$$

The currents i_{a}, i_{b}, and i_{c} are functions of time and are defined by Eqs. 5.1, 5.2, and 5.3, and thus

$$\begin{aligned} F(\theta, t) = {} & NI_{\mathrm{m}} \cos\omega t \cos\theta \\ & + NI_{\mathrm{m}} \cos(\omega t - 120°) \cos(\theta - 120°) \\ & + NI_{\mathrm{m}} \cos(\omega t + 120°) \cos(\theta + 120°) \end{aligned} \tag{5.16}$$

Using the trigonometric identity

$$\cos A \cos B = \tfrac{1}{2}\cos(A - B) + \tfrac{1}{2}\cos(A + B) \tag{5.17}$$

each term on the right-hand side of Eq. 5.16 can be expressed as the sum of two cosine functions, one involving the difference and the other the sum of the two angles. Therefore,

$$\begin{aligned} F(\theta, t) = {} & \tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t - \theta) + \tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t + \theta) \\ & + \tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t - \theta) + \tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t + \theta - 240°) \\ & + \underbrace{\tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t - \theta)}_{\text{Forward-rotating components}} + \underbrace{\tfrac{1}{2} NI_{\mathrm{m}} \cos(\omega t + \theta + 240°)}_{\text{Backward-rotating components}} \end{aligned} \tag{5.18}$$

$$= \tfrac{3}{2} NI_{\mathrm{m}} \cos(\omega t - \theta) \tag{5.19}$$

The expression of Eq. 5.19 represents the resultant mmf wave in the air gap. This represents an mmf rotating at the constant angular velocity $\omega\,(=2\pi f)$. At any instant of time, say t_1, the wave is distributed sinusoidally around the air gap (Fig. 5.6*b*) with the positive peak acting along $\theta=\omega t_1$. At a later instant, say t_2, the positive peak of the sinusoidally distributed wave is along $\theta=\omega t_2$; that is, the wave has moved by $\omega(t_2-t_1)$ around the air gap.

The angular velocity of the rotating mmf wave is $\omega=2\pi f$ radians per second and its rpm for a p-pole machine is given by Eq. 5.10.

It can be shown in general that an m-phase distributed winding excited by balanced m-phase currents will produce a sinusoidally distributed rotating field of constant amplitude when the phase windings are wound $2\pi/m$ electrical degrees apart in space. Note that a rotating magnetic field is produced without physically rotating any magnet. All that is necessary is to pass a polyphase current (ac) through the polyphase windings of the machine.

5.3 INDUCED VOLTAGES

In the preceding section it was shown that when balanced polyphase currents flow through a polyphase distributed winding, a sinusoidally distributed magnetic field rotates in the air gap of the machine. This effect can be visualized as one produced by a pair of magnets, for a two-pole machine, rotating in the air gap, the magnetic field (i.e., flux density) being sinusoidally distributed with the peak along the center of the magnetic poles. The result is illustrated in Fig. 5.7. The rotating field will induce voltages in the phase coils aa′, bb′, and cc′. Expressions for the induced voltages can be obtained by using Faraday's laws of induction.

The flux density distribution in the air gap can be expressed as

$$B(\theta)=B_{\max}\cos\theta \tag{5.20}$$

The air gap flux per pole, Φ_p, is

$$\Phi_\mathrm{p}=\int_{-\pi/2}^{\pi/2} B(\theta) lr\,d\theta = 2B_{\max}\,lr \tag{5.21}$$

where l is the axial length of the stator
r is the radius of the stator at the air gap

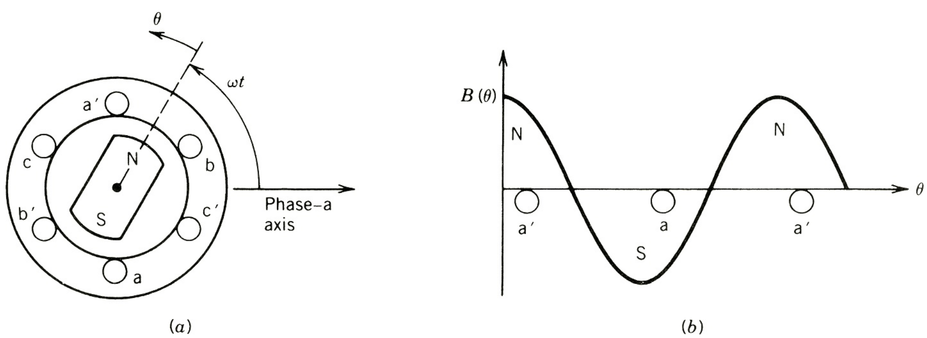


FIGURE 5.7 Air gap flux density distribution.

Let us consider that the phase coils are full-pitch coils of N turns (the coil sides of each phase are 180 electrical degrees apart as shown in Fig. 5.7). It is obvious that as the rotating field moves (or the magnetic poles rotate) the flux linkage of a coil will vary. The flux linkage for coil aa′ will be maximum ($=N\Phi_p$) at $\omega t = 0°$ (Fig. 5.7*a*) and zero at $\omega t = 90°$. The flux linkage $\lambda_a(\omega t)$ will vary as the cosine of the angle ωt. Hence,

$$\lambda_a(\omega t) = N\Phi_p \cos \omega t \tag{5.22}$$

Therefore, the voltage induced in phase coil aa′ is obtained from Faraday's law as

$$e_a = -\frac{d\lambda_a}{dt} = \omega N\Phi_p \sin \omega t \tag{5.23}$$

$$= E_{max} \sin \omega t \tag{5.23a}$$

The voltages induced in the other phase coils are also sinusoidal, but phase-shifted from each other by 120 electrical degrees. Thus,

$$e_b = E_{max} \sin(\omega t - 120°) \tag{5.24}$$

$$e_c = E_{max} \sin(\omega t + 120°) \tag{5.25}$$

From Eq. 5.23, the rms value of the induced voltage is

$$E_{rms} = \frac{\omega N\Phi_p}{\sqrt{2}} = \frac{2\pi f}{\sqrt{2}} N\Phi_p$$

$$= 4.44 f N\Phi_p \tag{5.26}$$

where f is the frequency in hertz. Equation 5.26 has the same form as that for the induced voltage in transformers (Eq. 1.40). However, Φ_p in Eq. 5.26 represents the flux per pole of the machine.

Equation 5.26 shows the rms voltage per phase. The N is the total number of series turns per phase, with the turns forming a concentrated full-pitch winding. In an actual ac machine, each phase winding is distributed in a number of slots (see Appendix A) for better use of the iron and copper and to improve the waveform. For such a distributed winding, the emfs induced in various coils placed in different slots are not in time phase, and therefore the phasor sum of the emfs is less than their numerical sum when they are connected in series for the phase winding. A reduction factor K_W, called the winding factor, must therefore be applied. For most three-phase machine windings, K_W is about 0.85 to 0.95. Therefore, for a distributed phase winding, the rms voltage per phase is

$$E_{rms} = 4.44 f N_{ph} \Phi_p K_W \tag{5.27}$$

where N_{ph} is the number of turns in series per phase.

5.4 POLYPHASE INDUCTION MACHINE

We shall now consider the various modes of operation of a polyphase induction machine. We shall first consider the standstill behavior of the machine, and then study its behavior in the running condition.

5.4.1 STANDSTILL OPERATION

Let us consider a three-phase wound-rotor induction machine with the rotor circuit left open-circuited. If the three-phase stator windings are connected to a three-phase supply, a rotating magnetic field will be produced in the air gap. The field rotates at synchronous speed n_s given by Eq. 5.10. This rotating field induces voltages in both stator and rotor windings at the same frequency f_1. The magnitudes of these voltages, from Eq. 5.27, are

$$E_1 = 4.44 f_1 N_1 \Phi_p K_{W1} \tag{5.28}$$

$$E_2 = 4.44 f_1 N_2 \Phi_p K_{W2} \tag{5.29}$$

Therefore,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}\frac{K_{W1}}{K_{W2}} \tag{5.30}$$

The winding factors K_{W1} and K_{W2} for the stator and rotor windings are normally the same. Thus,

$$\frac{E_1}{E_2} \simeq \frac{N_1}{N_2} = \text{turns ratio} \tag{5.31}$$

5.4.2 PHASE SHIFTER

Notice that the rotor can be held in such a position that the axes of the corresponding phase windings in the stator and the rotor make an angle β (Fig. 5.8*a*). In such a case, the induced

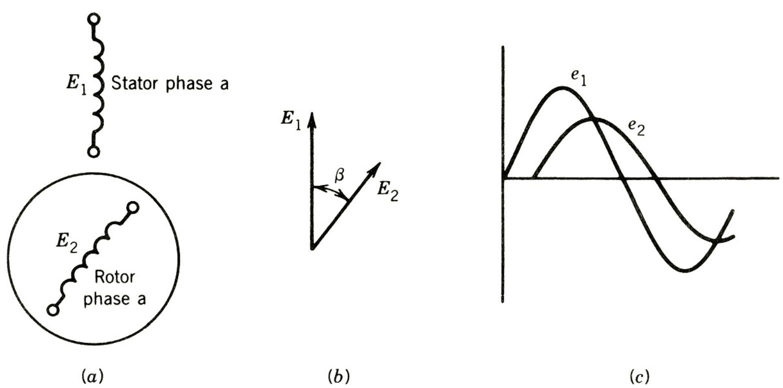


FIGURE 5.8 Induction machine as phase shifter.

voltage in the rotor winding will be phase-shifted from that of the stator winding by the angle β (Figs. 5.8*b* and 5.8*c*). Thus, a stationary wound-rotor induction machine can be used as a phase shifter. By turning the rotor mechanically, a phase shift range of 360° can be achieved.

5.4.3 INDUCTION REGULATOR

The stationary polyphase induction machine can also be used as a source of variable polyphase voltage if it is connected as an induction regulator as shown in Fig. 5.9*a*. The phasor diagram is shown in Fig. 5.9*b* to illustrate the principle. As the rotor is rotated through 360°, the output voltage V_o follows a circular locus of variable magnitude. If the induced voltages E_1 and E_2 are of the same magnitude (i.e., identical stator and rotor windings), the output voltage may be adjusted from zero to twice the supply voltage.

The induction regulator has the following advantages over a variable autotransformer:

- A continuous stepless variation of the output voltage is possible.
- No sliding electrical connections are necessary.

However, the induction regulator suffers from the disadvantages of higher leakage inductances, higher magnetizing current, and higher costs.

5.4.4 RUNNING OPERATION

If the stator windings are connected to a three-phase supply and the rotor circuit is closed, the induced voltages in the rotor windings produce rotor currents that interact with the air gap field to produce torque. The rotor, if free to do so, will then start rotating. According to Lenz's law, the rotor rotates in the direction of the rotating field such that the relative speed between the rotating field and the rotor winding decreases. The rotor will eventually reach a steady-state speed n that is less than the synchronous speed n_s at which the stator rotating field rotates in the air gap. It is obvious that at $n = n_s$ there will be no induced voltage and current in the rotor circuit, and hence no torque.

The difference between the rotor speed n and the synchronous speed n_s of the rotating field is called the *slip s* and is defined as

$$s = \frac{n_s - n}{n_s} \tag{5.32}$$

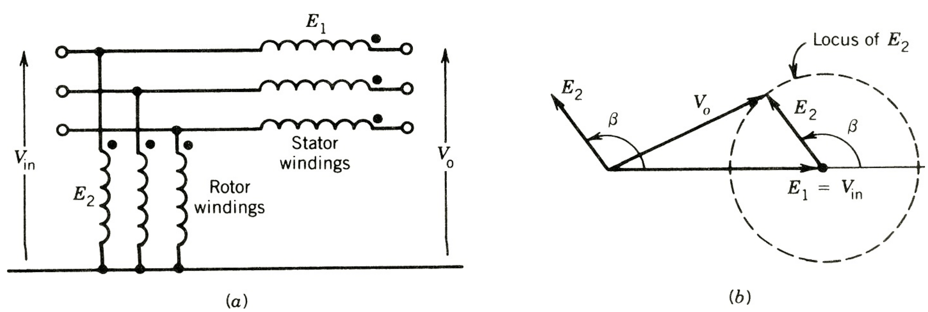


FIGURE 5.9 Induction regulator.

If you were sitting on the rotor, you would find that the rotor was slipping behind the rotating field by the *slip rpm* $= n_s - n = sn_s$. The frequency f_2 of the induced voltage and current in the rotor circuit will correspond to this slip rpm, because this is the relative speed between the rotating field and the rotor winding. Thus, from Eq. 5.10,

$$\begin{aligned} f_2 &= \frac{p}{120}(n_s - n) \\ &= \frac{p}{120} sn_s \\ &= sf_1 \end{aligned} \tag{5.33}$$

This rotor circuit frequency f_2 is also called *slip frequency*. The voltage induced in the rotor circuit at slip s is

$$\begin{aligned} E_{2s} &= 4.44 f_2 N_2 \Phi_p K_{W2} \\ &= 4.44 s f_1 N_2 \Phi_p K_{W2} \\ &= sE_2 \end{aligned} \tag{5.34}$$

where E_2 is the induced voltage in the rotor circuit at standstill—that is, at the stator frequency f_1.

The induced currents in the three-phase rotor windings also produce a rotating field. Its speed (rpm) n_2 with respect to the rotor is

$$\begin{aligned} n_2 &= \frac{120 f_2}{p} \\ &= \frac{120 s f_1}{p} \\ &= sn_s \end{aligned} \tag{5.35}$$

Because the rotor itself is rotating at n rpm, the induced rotor field rotates in the air gap at speed $n + n_2 = (1 - s)n_s + sn_s = n_s$ rpm. Therefore, both the stator field and the induced rotor field rotate in the air gap at the same synchronous speed n_s. The stator magnetic field and the rotor magnetic field are therefore stationary with respect to each other. The interaction between these two fields can be considered to produce the torque. As the magnetic fields tend to align, the stator magnetic field can be visualized as dragging the rotor magnetic field.

EXAMPLE 5.1

A 3ϕ, 460 V, 100 hp, 60 Hz, four-pole induction machine delivers rated output power at a slip of 0.05. Determine the

(a) Synchronous speed and motor speed.
(b) Speed of the rotating air gap field.
(c) Frequency of the rotor circuit.

(d) Slip rpm.

(e) Speed of the rotor field relative to the

(i) rotor structure.

(ii) stator structure.

(iii) stator rotating field.

(f) Rotor induced voltage at the operating speed, if the stator-to-rotor turns ratio is 1:0.5.

Solution

(a)

$$n_s = \frac{120f}{p} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$n = (1 - s)n_s = (1 - 0.05)1800 = 1710 \text{ rpm}$$

(b) 1800 rpm (same as synchronous speed)

(c) $f_2 = sf_1 = 0.05 \times 60 = 3 \text{ Hz}$

(d) slip rpm $= sn_s = 0.05 \times 1800 = 90$ rpm

(e) **(i)** 90 rpm

(ii) 1800 rpm

(iii) 0 rpm

(f) Assume that the induced voltage in the stator winding is the same as the applied voltage. Now,

$$\begin{aligned} E_{2s} &= sE_2 \\ &= s\frac{N_2}{N_1}E_1 \\ &= 0.05 \times 0.5 \times \frac{460 \text{ V}}{\sqrt{3}} \\ &= 6.64 \text{ V/phase} \quad \blacksquare \end{aligned}$$

5.5 THREE MODES OF OPERATION

The induction machine can be operated in three modes: motoring, generating, and plugging. To illustrate these three modes of operation, consider an induction machine mechanically coupled to a dc machine, as shown in Fig. 5.10*a*.

5.5.1 MOTORING

If the stator terminals are connected to a three-phase supply, the rotor will rotate in the direction of the stator rotating magnetic field. This is the natural (or motoring) mode of

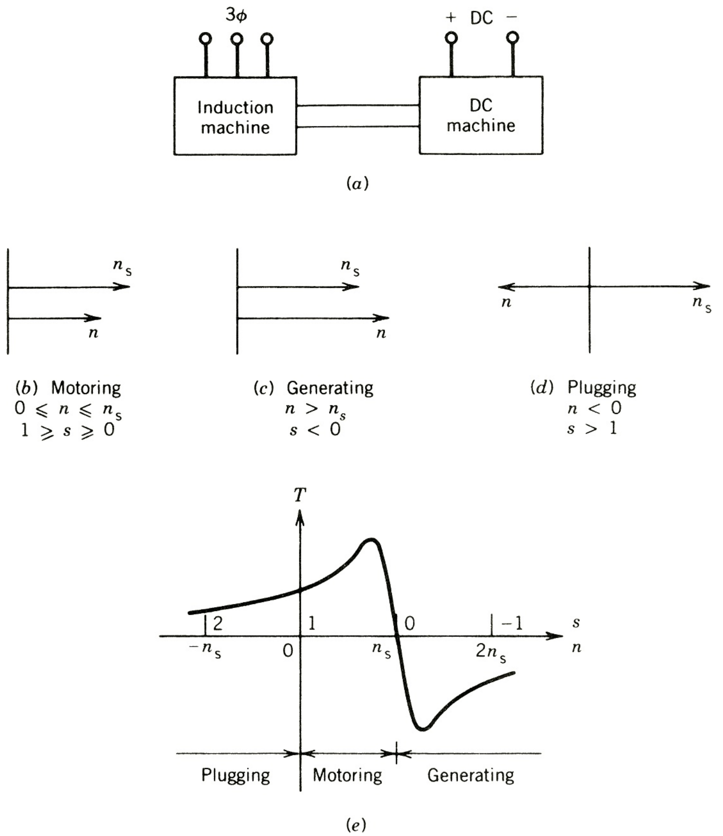


FIGURE 5.10 Three modes of operation of an induction machine.

operation of the induction machine. The steady-state speed n is less than the synchronous speed n_s, as shown in Fig. 5.10*b*.

5.5.2 GENERATING

The dc motor can be adjusted so that the speed of the system is higher than the synchronous speed and the system rotates in the same direction as the stator rotating field as shown in Fig. 5.10*c*. The induction machine will produce a generating torque—that is, a torque acting opposite to the rotation of the rotor (or acting opposite to the stator rotating magnetic field). The generating mode of operation is utilized in some drive applications to provide regenerative braking. For example, suppose an induction machine is fed from a variable-frequency supply to control the speed of a drive system. To stop the drive system, the frequency of the supply is gradually reduced. In the process, the instantaneous speed of the drive system is higher than the instantaneous synchronous speed because of the inertia of the drive system. As a result, the generating action of the induction machine will cause the power flow to reverse and the kinetic energy of the drive system will be fed back to the supply. The process is known as regenerative braking.

5.5.3 PLUGGING

If the dc motor of Fig. 5.10*a* is adjusted so that the system rotates in a direction opposite to the stator rotating magnetic field (Fig. 5.10*d*), the torque will be in the direction of the rotating field but will oppose the motion of the rotor. This torque is a braking torque.

This mode of operation is sometimes utilized in drive applications where the drive system is required to stop very quickly. Suppose an induction motor is running at a steady-state speed. If its terminal phase sequence is changed suddenly, the stator rotating field will rotate opposite to the rotation of the rotor, producing the plugging operation. The motor will come to zero speed rapidly and will accelerate in the opposite direction, unless the supply is disconnected at zero speed.

The three modes of operation and the typical torque profile of the induction machine in the various speed ranges are illustrated in Fig. 5.10*e*.

5.6 INVERTED INDUCTION MACHINE

In a wound-rotor induction machine the three-phase supply can be connected to the rotor windings through the slip rings and the stator terminals can be shorted, as shown in Fig. 5.11. A rotor-fed induction machine is also known as an inverted induction machine. The three-phase rotor current will produce a rotating field in the air gap, which will rotate at the synchronous speed with respect to the rotor. If the rotor is held stationary, this rotating field will also rotate in the air gap at the synchronous speed. Voltage and current will be induced in the stator windings and a torque will be developed. If the rotor is allowed to move, it will rotate, according to Lenz's law, opposite to the rotation of the rotating field so that the induced voltage in the stator winding is decreased. At a particular speed, therefore, the frequency of the stator circuit will correspond to the slip rpm.

5.7 EQUIVALENT CIRCUIT MODEL

The preceding sections have provided an appreciation of the physical behavior of the induction machine. We now proceed to develop an equivalent circuit model that can be used to study and predict the performance of the induction machine with reasonable accuracy. In this section, a steady-state per-phase equivalent circuit will be derived.

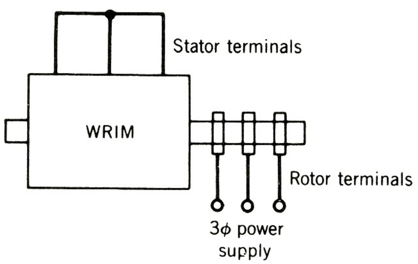


FIGURE 5.11 Inverted wound-rotor induction machine.

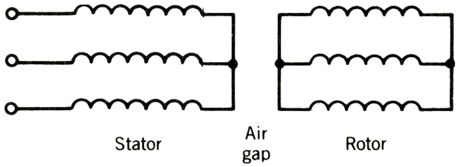


FIGURE 5.12 Three-phase wound-rotor induction motor.

For convenience, consider a three-phase wound-rotor induction machine as shown in Fig. 5.12. In the case of a squirrel-cage rotor, the rotor circuit can be represented by an equivalent three-phase rotor winding. If currents flow in both stator and rotor windings, rotating magnetic fields will be produced in the air gap. Because they rotate at the same speed in the air gap, they will produce a resultant air gap field rotating at the synchronous speed. This resultant air gap field will induce voltages in both stator windings (at the supply frequency f_1) and rotor windings (at slip frequency f_2). It appears that the equivalent circuit may assume a form identical to that of a transformer.

5.7.1 STATOR WINDING

The stator winding can be represented as shown in Fig. 5.13*a*, where

V_1 = per-phase terminal voltage
R_1 = per-phase stator winding resistance
L_1 = per-phase stator leakage inductance
E_1 = per-phase induced voltage in the stator winding
L_m = per-phase stator magnetizing inductance
R_c = per-phase stator core loss resistance

Note that there is no difference in form between this equivalent circuit and that of the transformer primary winding. The difference lies only in the magnitude of the parameters. For example, the excitation current I_ϕ is considerably larger in the induction machine because of the air gap. In induction machines it is as high as 30 to 50 percent of the rated current, depending on the motor size, whereas it is only 1 to 5 percent in transformers. Moreover, the leakage reactance X_1 is larger because of the air gap, and also because the stator and rotor windings are distributed along the periphery of the air gap rather than concentrated on a core, as in the transformer.

5.7.2 ROTOR CIRCUIT

The rotor equivalent circuit at slip s is shown in Fig. 5.13*b*, where

E_2 = per-phase induced voltage in rotor at standstill (i.e., at stator frequency f_1)
R_2 = per-phase rotor circuit resistance
L_2 = per-phase rotor leakage inductance

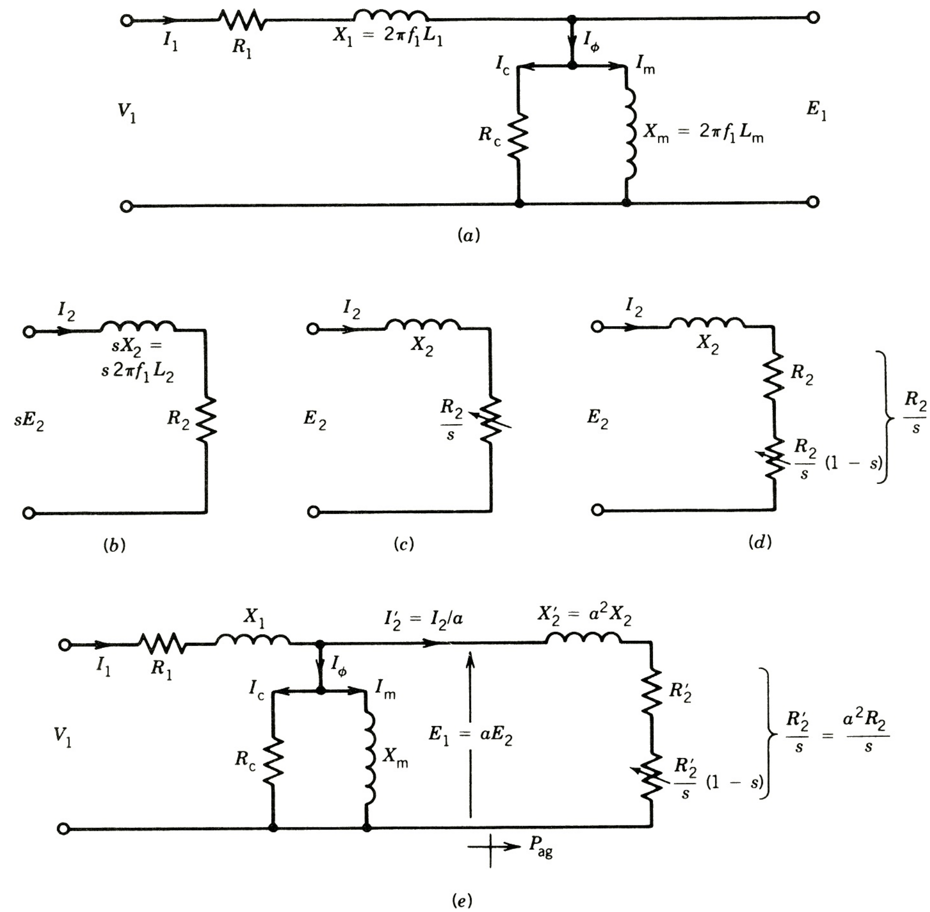


FIGURE 5.13 Development of the induction machine equivalent circuit.

Note that this circuit is at frequency f_2. The rotor current I_2 is

$$I_2 = \frac{sE_2}{R_2 + jsX_2} \tag{5.36}$$

The power involved in the rotor circuit is

$$P_2 = I_2^2 R_2 \tag{5.37}$$

which represents the rotor copper loss per phase.

Equation 5.36 can be rewritten as

$$I_2 = \frac{E_2}{(R_2/s) + jX_2} \tag{5.38}$$

Equation 5.38 suggests the rotor equivalent circuit of Fig. 5.13*c*. Although the magnitude and phase angle of I_2 are the same in Eqs. 5.36 and 5.38, there is a significant difference between these two equations and the circuits (Figs. 5.13*b* and 5.13*c*) they represent. The current I_2 in Eq. 5.36 is at slip frequency f_2, whereas I_2 in Eq. 5.38 is at line frequency f_1. In Eq. 5.36 the rotor leakage reactance sX_2 varies with speed but resistance R_2 remains fixed, whereas in Eq. 5.38 the resistance R_2/s varies with speed but the leakage reactance X_2 remains unaltered. The per-phase power associated with the equivalent circuit of Fig. 5.13*c* is

$$P = I_2^2 \frac{R_2}{s} = \frac{P_2}{s} \tag{5.39}$$

Because induction machines are operated at low slips (typical values of slip s are 0.01 to 0.05), the power associated with Fig. 5.13*c* is considerably larger. Note that the equivalent circuit of Fig. 5.13*c* is at the stator frequency, and therefore this is the rotor equivalent circuit as seen from the stator. The power in Eq. 5.39 therefore represents the power that crosses the air gap and thus includes the rotor copper loss as well as the mechanical power developed. Equation 5.39 can be rewritten as

$$P = P_{ag} = I_2^2 \left[R_2 + \frac{R_2}{s}(1-s) \right] \tag{5.40}$$

$$= I_2^2 \frac{R_2}{s} \tag{5.40a}$$

The corresponding equivalent circuit is shown in Fig. 5.13*d*. The speed-dependent resistance $R_2(1-s)/s$ represents the mechanical power developed by the induction machine.

$$\begin{aligned} P_{mech} &= I_2^2 \frac{R_2}{s}(1-s) \\ &= (1-s)P_{ag} \end{aligned} \tag{5.41}$$

$$= \frac{1-s}{s} P_2 \tag{5.42}$$

and

$$P_2 = I_2^2 R_2 = sP_{ag} \tag{5.43}$$

Thus,

$$P_{ag} : P_2 : p_{mech} = 1 : s : 1-s \tag{5.44}$$

Equation 5.44 indicates that of the total power input to the rotor (i.e., power crossing the air gap, P_{ag}), a fraction s is dissipated in the resistance of the rotor circuit (known as rotor copper loss), and the fraction $1-s$ is converted into mechanical power. Therefore, for efficient operation of the induction machine, it should operate at a low slip so that more of the air gap power is converted into mechanical power. Part of the mechanical power will be lost to overcome the windage and friction. The remainder of the mechanical power will be available as output shaft power.

5.7.3 COMPLETE EQUIVALENT CIRCUIT

The stator equivalent circuit, Fig. 5.13*a*, and the rotor equivalent circuit of Fig. 5.13*c* or 5.13*d* are at the same line frequency f_1 and therefore can be joined together. However, E_1 and E_2 may be different if the turns in the stator winding and the rotor winding are different. If the turns ratio ($a = N_1/N_2$) is considered, the equivalent circuit of the induction machine is that shown in Fig. 5.13*e*. Note that the form of the equivalent circuit is identical to that of a two-winding transformer, as expected.

EXAMPLE 5.2

A 3ϕ, 15 hp, 460 V, four-pole, 60 Hz, 1728 rpm induction motor delivers full output power to a load connected to its shaft. The windage and friction loss of the motor is 750 W. Determine the

(a) Mechanical power developed.
(b) Air gap power.
(c) Rotor copper loss.

Solution

(a) Full-load shaft power $= 15 \times 746 = 11{,}190$ W

$$\text{Mechanical power developed} = \text{shaft power} + \text{windage and friction loss}$$
$$= 11{,}190 + 750 = 11{,}940 \text{ W}$$

(b)

$$\text{Synchronous speed} \quad n_s = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$\text{Slip} \quad s = \frac{1800 - 1728}{1800} = 0.04$$

$$\text{Air gap power} \quad P_{ag} = \frac{11{,}940}{1 - 0.04} = 12{,}437.5 \text{ W}$$

(c)

$$\text{Rotor copper loss} \quad P_2 = 0.04 \times 12{,}437.5$$
$$= 497.5\, W \quad \blacksquare$$

5.7.4 VARIOUS EQUIVALENT CIRCUIT CONFIGURATIONS

The equivalent circuit shown in Fig. 5.13*e* is not convenient for predicting the performance of the induction machine. As a result, several simplified versions have been proposed in various textbooks on electric machines. There is no general agreement on how to treat the shunt branch (i.e., R_c and X_m), particularly the resistance R_c representing the core loss in the machine. Some of the commonly used versions of the equivalent circuit are discussed here.

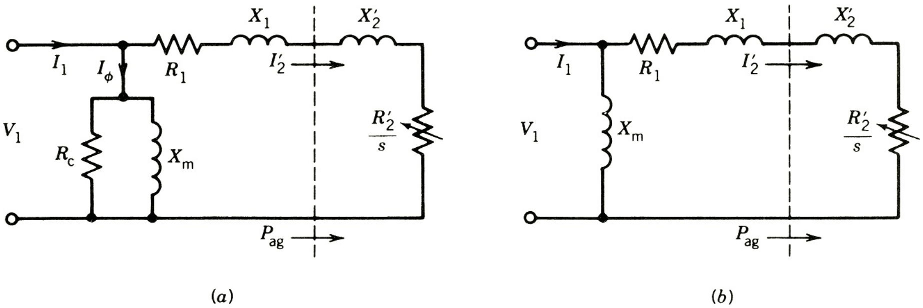


FIGURE 5.14 Approximate equivalent circuit.

Approximate Equivalent Circuit

If the voltage drop across R_1 and X_1 is small and the terminal voltage V_1 does not appreciably differ from the induced voltage E_1, the magnetizing branch (i.e., R_c and X_m) can be moved to the machine terminals as shown in Fig. 5.14*a*. This approximation of the equivalent circuit will considerably simplify computation, because the excitation current (I_ϕ) and the load component (I_2') of the machine current can be directly computed from the terminal voltage V_1 by dividing it by the corresponding impedances.

Note that if the induction machine is connected to a supply of fixed voltage and frequency, the stator core loss is fixed. At no load, the machine will operate close to synchronous speed. Therefore, the rotor frequency f_2 is very small and hence rotor core loss is very small. At a lower speed, f_2 increases and so does the rotor core loss. The total core losses thus increase as the speed falls. On the other hand, at no load, friction and windage losses are maximum, and as speed falls, these losses decrease. Therefore, if a machine operates from a constant-voltage and constant-frequency source, the sum of core losses and friction and windage losses remains essentially constant at all operating speeds. These losses can thus be lumped together and termed the constant *rotational losses* of the induction machine. If the core loss is lumped with the windage and friction loss, R_c can be removed from the equivalent circuit, as shown in Fig. 5.14*b*.

IEEE-Recommended Equivalent Circuit

In the induction machine, because of its air gap, the exciting current I_ϕ is high—of the order of 30 to 50 percent of the full-load current. The leakage reactance X_1 is also high. The IEEE recommends that in such a situation, the magnetizing reactance X_m not be moved to the machine terminals (as is done in Fig. 5.14*b*) but be retained at its appropriate place, as shown in Fig. 5.15. The resistance R_c is, however, omitted, and the core loss is lumped with the

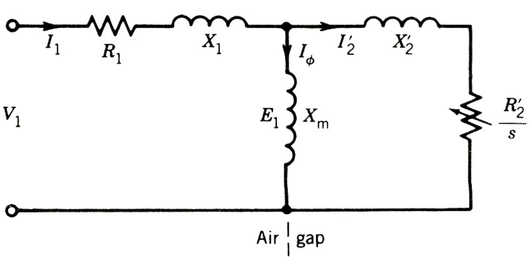


FIGURE 5.15 IEEE-recommended equivalent circuit.

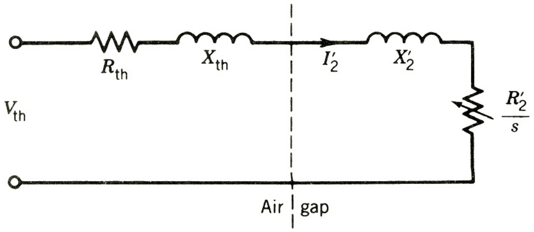


FIGURE 5.16 Thevenin equivalent circuit.

windage and friction losses. This equivalent circuit (Fig. 5.15) is to be preferred for situations in which the induced voltage E_1 differs appreciably from the terminal voltage V_1.

5.7.5 THEVENIN EQUIVALENT CIRCUIT

In order to simplify computations, V_1, R_1, X_1, and X_m can be replaced by the Thevenin equivalent circuit values V_{th}, R_{th}, and X_{th}, as shown in Fig. 5.16, where

$$V_{th} = \frac{X_m}{[R_1^2 + (X_1 + X_m)^2]^{1/2}} V_1 \tag{5.45}$$

If $R_1^2 \ll (X_1 + X_m)^2$, as is usually the case,

$$V_{th} \approx \frac{X_m}{X_1 + X_m} \tag{5.45a}$$

$$= K_{th} V_1 \tag{5.45b}$$

The Thevenin impedance is

$$Z_{th} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

$$= R_{th} + jX_{th}$$

If $R_1^2 \ll (X_1 + X_m)^2$,

$$R_{th} \simeq \left(\frac{X_m}{X_1 + X_m}\right)^2 R_1 \tag{5.46}$$

$$= K_{th}^2 R_1 \tag{5.46a}$$

and since $X_1 \ll X_m$,

$$X_{th} \simeq X_1 \tag{5.47}$$

5.8 NO-LOAD TEST, BLOCKED-ROTOR TEST, AND EQUIVALENT CIRCUIT PARAMETERS

The parameters of the equivalent circuit, R_c, X_m, R_1, X_1, X_2, and R_2, can be determined from the results of a no-load test, a blocked-rotor test, and measurement of the dc resistance of the stator winding. The no-load test on an induction machine, like the open-circuit test on a transformer, gives information about exciting current and rotational losses. This test is performed by applying balanced polyphase voltages to the stator windings at the rated frequency. The rotor is kept uncoupled from any mechanical load. The small power loss in the machine at no load is due to the core loss and the friction and windage loss. The total rotational loss at the rated voltage and frequency under load is usually considered to be constant and equal to its value at no load.

The blocked-rotor test on an induction machine, like the short-circuit test on a transformer, gives information about leakage impedances. In this test the rotor is blocked so that the motor cannot rotate, and balanced polyphase voltages are applied to the stator terminals. The blocked-rotor test should be performed under the same conditions of rotor current and frequency that will prevail in the normal operating conditions. For example, if the performance characteristics in the normal running condition (i.e., low-slip region) are required, the blocked-rotor test should be performed at a reduced voltage and rated current. The frequency also should be reduced because the rotor effective resistance and leakage inductance at the reduced frequency (corresponding to lower values of slip) may differ appreciably from their values at the rated frequency. This will be particularly true for double-cage or deep-bar rotors, as discussed in Section 5.11, and also for high-power motors.

The IEEE recommends a frequency of 25 percent of the rated frequency for the blocked-rotor test. The leakage reactances at the rated frequency can then be obtained by considering that the reactance is proportional to frequency. However, for normal motors of less than 20-hp rating, the effects of frequency are negligible, and the blocked-rotor test can be performed directly at the rated frequency.

The determination of the equivalent circuit parameters from the results of the no-load and blocked-rotor tests is illustrated by the following example.

EXAMPLE 5.3

The following test results are obtained from a 3ϕ, 60 hp, 2200 V, six-pole, 60 Hz squirrel-cage induction motor.

1. No-load test:

$$\text{Supply frequency} = 60\ \text{Hz}$$
$$\text{Line voltage} = 2200\ \text{V}$$
$$\text{Line current} = 4.5\ \text{A}$$
$$\text{Input power} = 1600\ \text{W}$$

2. Blocked-rotor test:

$$\text{Frequency} = 15\text{ Hz}$$

$$\text{Line voltage} = 270\text{ V}$$

$$\text{Line current} = 25\text{ A}$$

$$\text{Input power} = 9000\text{ W}$$

3. Average dc resistance per stator phase:

$$R_1 = 2.8\,\Omega$$

- **(a)** Determine the no-load rotational loss.
- **(b)** Determine the parameters of the IEEE-recommended equivalent circuit of Fig. 5.15.
- **(c)** Determine the parameters (V_{th}, R_{th}, X_{th}) for the Thevenin equivalent circuit of Fig. 5.16.

Solution

(a) From the no-load test, the no-load power is

$$P_{NL} = 1600\text{ W}$$

The no-load rotational loss is

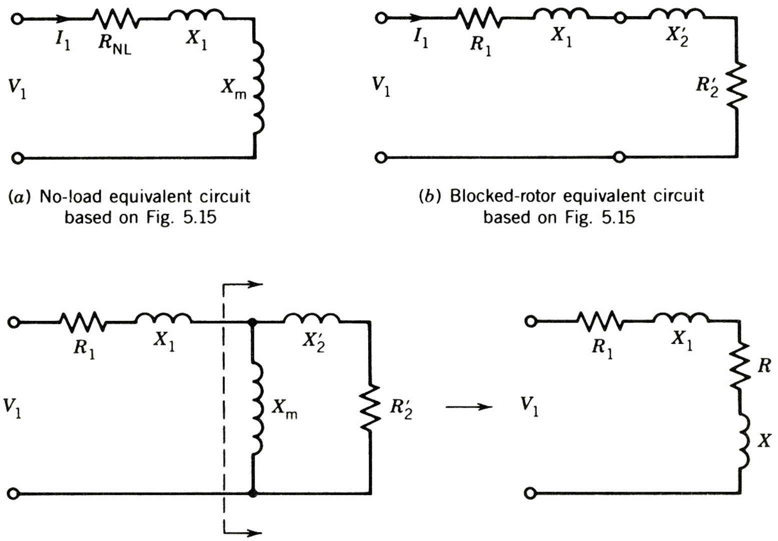
$$\begin{aligned} P_{Rot} &= P_{NL} - 3I_1^2R_1 \\ &= 1600 - 3 \times 4.5^2 \times 2.8 \\ &= 1429.9\text{ W} \end{aligned}$$

(b) *IEEE-recommended equivalent circuit*. For the no-load condition, R_2'/s is very high. Therefore, in the equivalent circuit of Fig. 5.15, the magnetizing reactance X_m is shunted by a very high resistive branch representing the rotor circuit. The reactance of this parallel combination is almost the same as X_m. Therefore, the total reactance X_{NL}, measured at no load at the stator terminals, is essentially $X_1 + X_m$. The equivalent circuit at no load is shown in Fig. E5.3*a*.

$$V_1 = \frac{2200}{\sqrt{3}} = 1270.2\text{ V/phase}$$

The no-load impedance is

$$Z_{NL} = \frac{V_1}{I_1} = \frac{1270.2}{4.5} = 282.27\,\Omega$$



(*a*) No-load equivalent circuit based on Fig. 5.15

(*b*) Blocked-rotor equivalent circuit based on Fig. 5.15

(*c*) Blocked-rotor equivalent circuit for improved value for R_2'

FIGURE E5.3

The no-load resistance is

$$R_{NL} = \frac{P_{NL}}{3I_1^2} = \frac{1600}{3 \times 4.5^2} = 26.34\,\Omega$$

The no-load reactance is

$$\begin{aligned} X_{NL} &= (Z_{NL}^2 - R_{NL}^2)^{1/2} \\ &= (282.27^2 - 26.34^2)^{1/2} \\ &= 281.0\,\Omega \end{aligned}$$

Thus, $X_1 + X_m = X_{NL} = 281.0\,\Omega$.

For the blocked-rotor test the slip is 1. In the equivalent circuit of Fig. 5.15, the magnetizing reactance X_m is shunted by the low-impedance branch $jX_2' + R_2'$. Because $|X_m| \gg |R_2' + jX_2'|$, the impedance X_m can be neglected and the equivalent circuit for the blocked-rotor test reduces to the form shown in Fig. E5.3*b*. From the blocked-rotor test, the blocked-rotor resistance is

$$\begin{aligned} R_{BL} &= \frac{P_{BL}}{3I_1^2} \\ &= \frac{9000}{3 \times 25^2} \\ &= 4.8\,\Omega \end{aligned}$$

Therefore, $R_2' = R_{BL} - R_1 = 4.8 - 2.8 = 2\,\Omega$. The blocked-rotor impedance at 15 Hz is

$$Z_{BL} = \frac{V_1}{I_1} = \frac{270}{\sqrt{3}\times 25} = 6.24\,\Omega$$

The blocked-rotor reactance at 15 Hz is

$$\begin{aligned} X_{BL} &= (6.24^2 - 4.8^2)^{1/2} \\ &= 3.98\,\Omega \end{aligned}$$

Its value at 60 Hz is

$$X_{BL} = 3.98 \times \frac{60}{15} = 15.92\,\Omega$$

$$X_{BL} = X_1 + X_2'$$

Hence,

$$X_1 = X_2' = \frac{15.92}{2} = 7.96\,\Omega \quad \text{(at 60 Hz)}$$

The magnetizing reactance is therefore

$$X_m = 281.0 - 7.96 = 273.04\,\Omega$$

Comments: The rotor equivalent resistance R_2' plays an important role in the performance of the induction machine. A more accurate determination of R_2' is recommended by the IEEE as follows: The blocked resistance R_{BL} is the sum of R_1 and an equivalent resistance, say R, which is the resistance of $R_2' + jX_2'$ in parallel with X_m as shown in Fig. E5.3*c*; therefore,

$$R = \frac{X_m^2}{R_2'^2 + (X_2' + X_m)^2} R_2'$$

If $X_2' + X_m \gg R_2'$, as is usually the case,

$$R \simeq \left(\frac{X_m}{X_2' + X_m}\right)^2 R_2'$$

or

$$R_2' = \left(\frac{X_2' + X_m}{X_m}\right)^2 R$$

Now $R = R_{BL} - R_1 = 4.8 - 2.8 = 2\,\Omega$. So,

$$R_2' = \left(\frac{7.96 + 273.04}{273.04}\right)^2 \times 2 = 2.12\,\Omega$$

(c) From Eq. 5.45,

$$V_{th} \simeq \frac{273.04}{7.96 + 273.04} V_1$$

$$= 0.97 V_1$$

From Eq. 5.46a,

$$R_{th} \simeq 0.97^2 R_1 = 0.97^2 \times 2.8 = 2.63\ \Omega$$

From Eq. 5.47,

$$X_{th} \simeq X_1 = 7.96\ \Omega \quad \blacksquare$$

5.9 PERFORMANCE CHARACTERISTICS

The equivalent circuits derived in the preceding section can be used to predict the performance characteristics of the induction machine. The important performance characteristics in the steady state are the efficiency, power factor, current, starting torque, maximum (or pull-out) torque, and so forth.

The mechanical torque developed T_{mech} per phase is given by

$$P_{mech} = T_{mech}\omega_{mech} = I_2^2 \frac{R_2}{s}(1 - s) \tag{5.48}$$

where

$$\omega_{mech} = \frac{2\pi n}{60} \tag{5.48a}$$

The mechanical speed ω_{mech} is related to the synchronous speed by

$$\omega_{mech} = (1 - s)\omega_{syn} \tag{5.49}$$

$$= \frac{n_{syn}}{60} 2\pi(1 - s)$$

and

$$\omega_{syn} = \frac{120f}{p60} \times 2\pi = \frac{4\pi f_1}{p} \tag{5.50}$$

From Eqs. 5.48, 5.49, and 5.40a,

$$T_{mech}\omega_{syn} = I_2^2 \frac{R_2}{s} = P_{ag} \tag{5.51}$$

$$T_{\text{mech}} = \frac{1}{\omega_{\text{syn}}} P_{\text{ag}} \tag{5.52}$$

$$= \frac{1}{\omega_{\text{syn}}} I_2^2 \frac{R_2}{s} \tag{5.52a}$$

$$= \frac{1}{\omega_{\text{syn}}} I_2'^2 \frac{R_2'}{s} \tag{5.53}$$

From the equivalent circuit of Fig. 5.16 and Eq. 5.53,

$$T_{\text{mech}} = \frac{1}{\omega_{\text{syn}}} \frac{V_{\text{th}}^2}{(R_{\text{th}} + R_2'/s)^2 + (X_{\text{th}} + X_2')^2} \frac{R_2'}{s} \tag{5.54}$$

Note that if the approximate equivalent circuits (Fig. 5.14) are used to determine I_2', in Eq. 5.54, V_{th}, R_{th}, and X_{th} should be replaced by V_1, R_1, and X_1, respectively. The prediction of performance based on the approximate equivalent circuit (Fig. 5.14) may differ by 5 percent from those based on the equivalent circuit of Fig. 5.15 or 5.16.

For a three-phase machine, Eq. 5.54 should be multiplied by 3 to obtain the total torque developed by the machine. The torque–speed characteristic is shown in Fig. 5.17. At low values of slip,

$$R_{\text{th}} + \frac{R_2'}{s} \gg X_{\text{th}} + X_2' \quad \text{and} \quad \frac{R_2'}{s} \gg R_{\text{th}}$$

and thus

$$T_{\text{mech}} \approx \frac{1}{\omega_{\text{syn}}} \frac{V_{\text{th}}^2}{R_2'} s \tag{5.55}$$

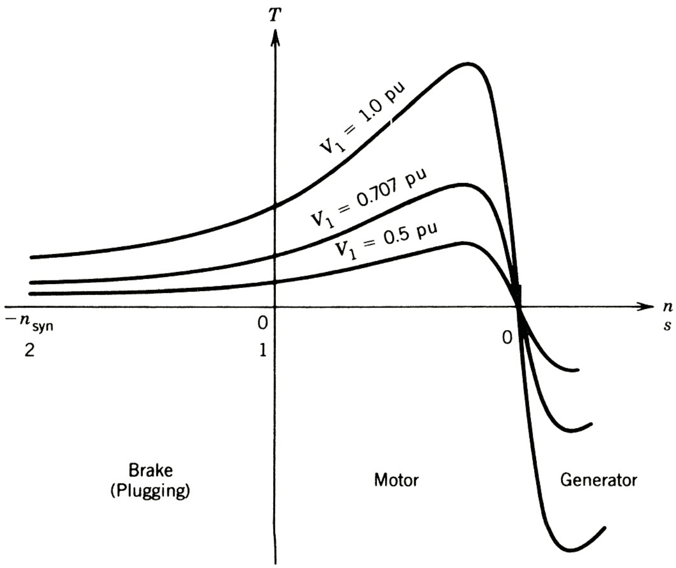


FIGURE 5.17 Torque–speed profile at different voltages.

The linear torque–speed relationship is evident in Fig. 5.17 near the synchronous speed. Note that if the approximate equivalent circuits (Fig. 5.14) are used, in Eq. 5.55, V_{th} should be replaced by V_1. At larger values of slip,

$$R_{th} + \frac{R_2'}{s} \ll X_{th} + X_2'$$

and

$$T_{mech} \approx \frac{1}{\omega_{syn}} \frac{V_{th}^2}{(X_{th} + X_2')^2} \frac{R_2'}{s} \tag{5.56}$$

The torque varies almost inversely with slip near $s = 1$, as seen from Fig. 5.17.

Equation 5.54 also indicates that at a particular speed (i.e., a fixed value of s) the torque varies as the square of the supply voltage V_{th} (hence V_1). Figure 5.17 shows the $T - n$ profile at various supply voltages. This aspect will be discussed further in a later section on speed control of induction machines by changing the stator voltage.

An expression for maximum torque can be obtained by setting $dT/ds = 0$. Differentiating Eq. 5.54 with respect to slip s and equating the result to zero gives the following condition for maximum torque:

$$\frac{R_2'}{s_{T_{max}}} = [R_{th}^2 + (X_{th} + X_2')^2]^{1/2} \tag{5.57}$$

This expression can also be derived from the fact that the condition for maximum torque corresponds to the condition for maximum air gap power (Eq. 5.52). This occurs, by the familiar impedance-matching principle in circuit theory, when the impedance of R_2'/s equals in magnitude the impedance between it and the supply voltage V_1 (Fig. 5.16) as shown in Eq. 5.57. The slip $s_{T_{max}}$ at maximum torque T_{max} is

$$s_{T_{max}} = \frac{R_2'}{[R_{th}^2 + (X_{th} + X_2')^2]^{1/2}} \tag{5.58}$$

The maximum torque per phase from Eqs. 5.54 and 5.58 is

$$T_{max} = \frac{1}{2\omega_{syn}} \frac{V_{th}^2}{R_{th} + [R_{th}^2 + (X_{th} + X_2')^2]^{1/2}} \tag{5.59}$$

Equation 5.59 shows that the maximum torque developed by the induction machine is independent of the rotor circuit resistance. However, from Eq. 5.58 it is evident that the value of the rotor circuit resistance R_2 determines the speed at which this maximum torque will occur. The torque–speed characteristics for various values of R_2 are shown in Fig. 5.18. In a wound-rotor induction motor, external resistance is added to the rotor circuit to make the maximum torque occur at standstill so that high starting torque can be obtained. As the motor speeds up, the external resistance is gradually decreased and finally taken out completely. Some induction motors are, in fact, designed so that maximum torque is available at start—that is, at zero speed.

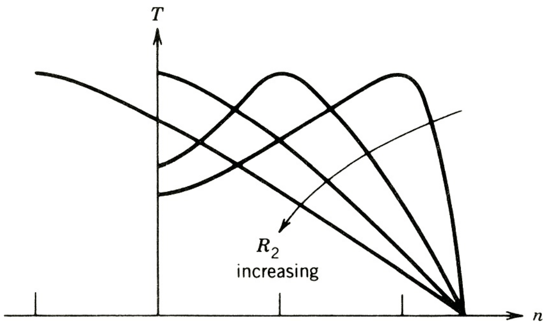


FIGURE 5.18 Torque–speed characteristics for varying R_2.

If the stator resistance R_1 is small (hence, R_{th} is negligibly small), from Eqs. 5.58 and 5.59,

$$s_{T_{max}} \simeq \frac{R_2'}{X_{th} + X_2'} \tag{5.60}$$

$$T_{max} \simeq \frac{1}{2\omega_{syn}} \frac{V_{th}^2}{X_{th} + X_2'} \tag{5.61}$$

Equation 5.61 indicates that the maximum torque developed by an induction machine is inversely proportional to the sum of the leakage reactances.

From Eq. 5.54, the ratio of the maximum developed torque to the torque developed at any speed is

$$\frac{T_{max}}{T} = \frac{(R_{th} + R_2'/s)^2 + (X_{th} + X_2')^2}{(R_{th} + R_2'/s_{T_{max}})^2 + (X_{th} + X_2')^2} \frac{s}{s_{T_{max}}} \tag{5.62}$$

If R_1 (hence R_{th}) is negligibly small,

$$\frac{T_{max}}{T} \simeq \frac{(R_2'/s)^2 + (X_{th} + X_2')^2}{(R_2'/s_{T_{max}})^2 + (X_{th} + X_2')^2} \frac{s}{s_{T_{max}}} \tag{5.63}$$

From Eqs. 5.60 and 5.63,

$$\frac{T_{max}}{T} = \frac{(R_2'/s)^2 + (R_2'/s_{T_{max}})^2}{2(R_2'/s_{T_{max}})^2} \times \frac{s}{s_{T_{max}}}$$

$$= \frac{s_{T_{max}}^2 + s^2}{2s_{T_{max}}s} \tag{5.64}$$

Equation 5.64 shows the relationship between torque at any speed and the maximum torque in terms of their slip values.

Stator Current

From Fig. 5.15, the input impedance is

$$Z_1 = R_1 + jX_1 + Xm /\!/ \left(\frac{R_2'}{s} + jX_2' \right)$$

$$= R_1 + jX_1 + X_{\mathrm{m}} /\!/ Z_2' \tag{5.65}$$

$$= R_1 + jX_1 + \frac{jX_{\mathrm{m}}(R_2'/s + jX_2')}{R_2'/s + j(X_{\mathrm{m}} + X_2')} \tag{5.65a}$$

$$= |Z_1| \angle \theta_1 \tag{5.65b}$$

The stator current is

$$I_1 = \frac{V_1}{Z_1} = I_\phi + I_2' \tag{5.65c}$$

At synchronous speed (i.e., $s = 0$), R_2'/s is infinite, so $I_2' = 0$. The stator current I_1 is the exciting current I_ϕ. At larger values of slip $Z_2' (= R_2'/s + jX_2')$ is low and therefore I_2' (and hence I_1) is large. In fact, the typical starting current (i.e., at $s = 1$) is five to eight times the rated current. The typical stator current variation with speed is shown in Fig. 5.19.

Input Power Factor

The supply power factor is given by

$$\mathrm{PF} = \cos \theta_1$$

where θ_1 is the phase angle of the stator current I_1. This phase angle θ_1 is the same as the impedance angle of the equivalent circuit of Fig. 5.15. The typical power factor variation with speed is shown in Fig. 5.20.

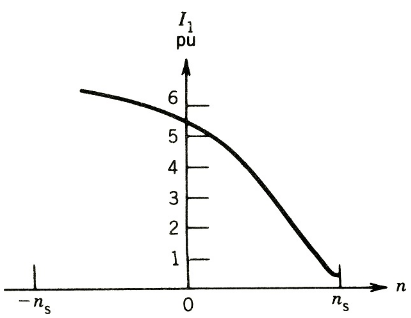


FIGURE 5.19 Stator current as a function of speed.

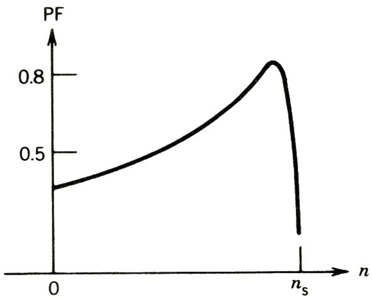


FIGURE 5.20 Power factor as a function of speed.

Efficiency

In order to determine the efficiency of the induction machine as a power converter, the various losses in the machine are first identified. These losses are illustrated in the power flow diagram of Fig. 5.21. For a 3ϕ machine, the power input to the stator is

$$P_{in} = 3\,V_1 I_1 \cos\theta_1 \tag{5.66}$$

The power loss in the stator windings is

$$P_1 = 3I_1^2 R_1 \tag{5.67}$$

where R_1 is the ac resistance (including skin effect) of each phase winding at the operating temperature and frequency.

Power is also lost as hysteresis and eddy current loss in the magnetic material of the stator core.

The remaining power, P_{ag}, crosses the air gap. Part of it is lost in the resistance of the rotor circuit.

$$P_2 = 3I_2^2 R_2 \tag{5.68}$$

where R_2 is the ac resistance of the rotor winding. If it is a wound-rotor machine, R_2 also includes any external resistance connected to the rotor circuit through slip rings.

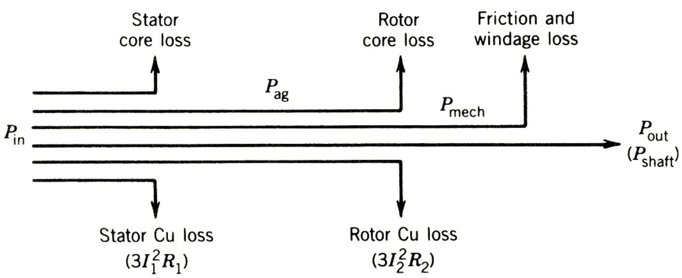


FIGURE 5.21 Power flow in an induction motor.

Power is also lost in the rotor core. Because the core losses are dependent on the frequency f_2 of the rotor, these may be negligible at normal operating speeds, where f_2 is very low.

The remaining power is converted into mechanical form. Part of this is lost as windage and friction losses, which are dependent on speed. The rest is the mechanical output power P_{out}, which is the useful power output from the machine.

The efficiency of the induction motor is

$$\text{Eff} = \frac{P_{\text{out}}}{P_{\text{in}}} \tag{5.69}$$

The efficiency is highly dependent on slip. If all losses are neglected except those in the resistance of the rotor circuit,

$$P_{\text{ag}} = P_{\text{in}}$$

$$P_2 = sP_{\text{ag}}$$

$$P_{\text{out}} = P_{\text{mech}} = P_{\text{ag}}(1 - s)$$

and the ideal efficiency is

$$\text{Eff}_{(\text{ideal})} = \frac{P_{\text{out}}}{P_{\text{in}}} = 1 - s \tag{5.70}$$

Sometimes $\text{Eff}_{(\text{ideal})}$ is also called the *internal efficiency*, as it represents the ratio of the power output to the air gap power. The ideal efficiency as a function of speed is shown in Fig. 5.22. It indicates that an induction machine must operate near its synchronous speed if high efficiency is desired. This is why the slip is very low for normal operation of the induction machine.

If other losses are included, the actual efficiency is lower than the ideal efficiency of Eq. 5.70, as shown in Fig. 5.22. The full-load efficiency of a large induction motor may be as high as 95 percent.

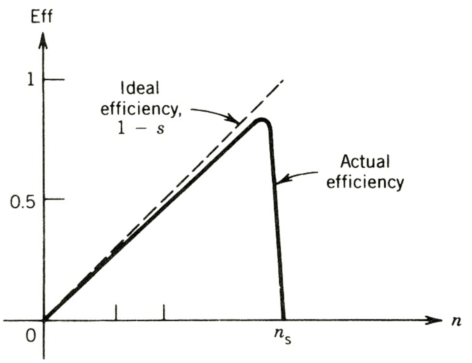


FIGURE 5.22 Efficiency as a function of speed.

5.10 POWER FLOW IN THREE MODES OF OPERATION

It was pointed out in Section 5.5 that the induction machine can be operated in three modes: motoring, generating, and plugging. The power flow in the machine will depend on the mode of operation. However, the equations derived in Section 5.9 for various power relationships hold good for all modes of operation. If the appropriate sign of the slip s is used in these expressions, the sign of the power will indicate the actual power flow. For example, in the generating mode, the slip is negative. Therefore, from Eq. 5.40a the air gap power P_{ag} is negative (note that the copper loss P_2 in the rotor circuit is always positive). This implies that the actual power flow across the air gap in the generating mode is from rotor to stator.

The power flow diagram in the three modes of operation is shown in Fig. 5.23. The core losses and the friction and windage losses are all lumped together as a constant rotational loss.

In the motoring mode, slip s is positive. The air gap power P_{ag} (Eq. 5.40a) and the developed mechanical power P_{mech} (Eq. 5.42) are positive, as shown in Fig. 5.23*a*.

In the generating mode s is negative, and therefore both P_{ag} and P_{mech} are negative, as shown in Fig. 5.23*b*. In terms of the equivalent circuit of Fig. 5.13*e* the resistance $[(1-s)/s]R_2$ is negative, which indicates that this resistance represents a source of energy.

In the plugging mode, s is greater than one, and therefore P_{ag} is positive but P_{mech} is negative, as shown in Fig. 5.23*c*. In this mode the rotor rotates opposite to the rotating field and therefore mechanical energy must be put into the system. Power therefore flows from both sides, and as a result the loss in the rotor circuit, P_2, is enormously increased. In terms of the equivalent circuit of Fig. 5.13*e*, the resistance $[(1-s)/s]R_2$ is negative and represents a source of energy.

Example 5.7 will further illustrate the power flow in the three modes of operation of an induction machine.

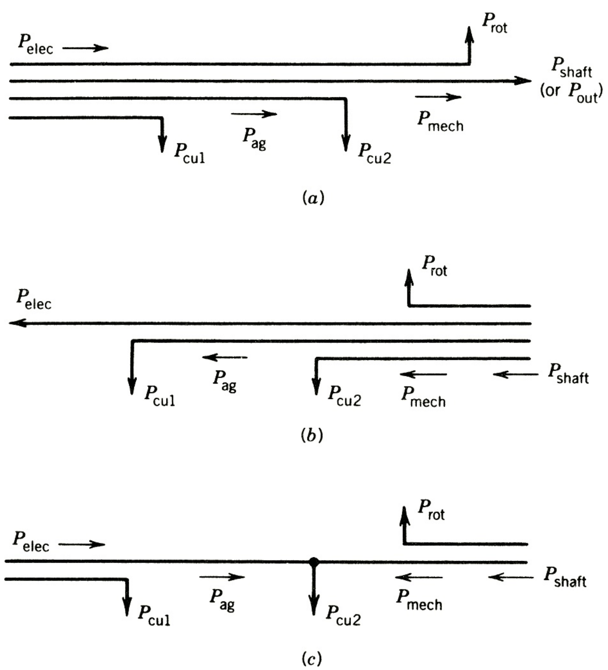


FIGURE 5.23 Power flow for various modes of operation of an induction machine. (*a*) Motoring mode, $0 < s < 1$. (*b*) Generating mode, $s < 0$. (*c*) Plugging mode, $s > 1$.

EXAMPLE 5.4

A three-phase, 460 V, 1740 rpm, 60 Hz, four-pole wound-rotor induction motor has the following parameters per phase:

$$R_1 = 0.25 \text{ ohms}, \qquad R_2' = 0.2 \text{ ohms}$$
$$X_1 = X_2' = 0.5 \text{ ohms}, \qquad X_m = 30 \text{ ohms}$$

The rotational losses are 1700 watts. With the rotor terminals short-circuited, find

(a) **(i)** Starting current when started direct on full voltage.
(ii) Starting torque.

(b) **(i)** Full-load slip.
(ii) Full-load current.
(iii) Ratio of starting current to full-load current.
(iv) Full-load power factor.
(v) Full-load torque.
(vi) Internal efficiency and motor efficiency at full load.

(c) **(i)** Slip at which maximum torque is developed.
(ii) Maximum torque developed.

(d) How much external resistance per phase should be connected in the rotor circuit so that maximum torque occurs at start?

Solution

(a)

$$V_1 = \frac{460}{\sqrt{3}} = 265.6 \text{ volts/phase}$$

(i) At start $s = 1$. The input impedance is

$$Z_1 = 0.25 + j0.5 + \frac{j30(0.2 + j0.5)}{0.2 + j30.5}$$
$$= 1.08\underline{/66^\circ} \ \Omega$$
$$I_{st} = \frac{265.6}{1.08\underline{/66^\circ}} = 245.9\underline{/-66^\circ} \text{ A}$$

(ii)

$$\omega_{syn} = \frac{1800}{60} \times 2\pi = 188.5 \text{ rad/sec}$$
$$V_{th} = \frac{265.6(j30.0)}{(0.25 + j30.5)} \approx 261.3 \text{ V}$$
$$Z_{th} = \frac{j30(0.25 + j0.5)}{0.25 + j30.5} = 0.55\underline{/63.9^\circ}$$
$$= 0.24 + j0.49$$

$$R_{\text{th}} = 0.24\ \Omega$$

$$X_{\text{th}} = 0.49 \simeq X_1$$

$$T_{\text{st}} = \frac{P_{\text{ag}}}{\omega_{\text{syn}}} = \frac{I_2'^2 R_2'/s}{\omega_{\text{syn}}}$$

$$= \frac{3}{188.5} \frac{261.3^2}{(0.24+0.2)^2 + (0.49+0.5)^2} \times \frac{0.2}{1}$$

$$= \frac{3}{188.5} \times (241.2)^2 \times \frac{0.2}{1}$$

$$s = 185.2\ \text{N}\cdot\text{m}$$

(b) **(i)** $$s = \frac{1800 - 1740}{1800} = 0.0333$$

(ii) $$\frac{R_2'}{s} = \frac{0.2}{0.0333} = 6.01\ \Omega$$

$$Z_1 = (0.25 + j0.5) + \frac{(j30)(6.01 + j0.5)}{6.01 + j30.5}$$

$$= 0.25 + j0.5 + 5.598 + j1.596$$

$$= 6.2123\underline{/19.7^\circ}\ \Omega$$

$$I_{\text{FL}} = \frac{265.6}{6.2123\underline{/19.7}}$$

$$= 42.754\underline{/-19.7^\circ}$$

(iii) $$\frac{I_{\text{st}}}{I_{\text{FL}}} = \frac{245.9}{42.754} = 5.75$$

(iv) $$\text{PF} = \cos(19.7^\circ) = 0.94\ \text{(lagging)}$$

(v) $$T = \frac{3}{188.5} \frac{(261.3)^2}{(0.24+6.01)^2 + (0.49+0.5)^2} \times 6.01$$

$$= \frac{3}{188.5} \times 41.29^2 \times 6.01$$

$$= 163.11\ \text{N}\cdot\text{m}$$

(vi) Air gap power:

$$P_{\text{ag}} = T\omega_{\text{syn}} = 163.11 \times 188.5 = 30{,}746.2\ \text{W}$$

Rotor copper loss:

$$P_2 = sP_{ag} = 0.0333 \times 30{,}746.2 = 1023.9\ \text{W}$$

$$P_{mech} = (1 - 0.0333)30{,}746.2 = 29{,}722.3\ \text{W}$$

$$P_{out} = P_{mech} - P_{rot} = 29{,}722.3 - 1700 = 28{,}022.3\ \text{W}$$

$$\begin{aligned} P_{input} &= 3V_1 I_1 \cos\theta \\ &= 3 \times 265.6 \times 42.754 \times 0.94 = 32{,}022.4\ \text{W} \end{aligned}$$

$$\text{Eff}_{motor} = \frac{28{,}022.3}{32{,}022.4} \times 100 = 87.5\%$$

$$\text{Eff}_{internal} = (1 - s) = 1 - 0.0333 = 0.967 \rightarrow 96.7\%$$

(c) **(i)** From Eq. 5.58,

$$s_{T_{max}} = \frac{0.2}{[0.24^2 + (0.49 + 0.5)^2]^{1/2}} = \frac{0.2}{1.0187} = 0.1963$$

(ii) From Eq. 5.59,

$$\begin{aligned} T_{max} &= \frac{3}{2 \times 188.5}\left[\frac{261.3^2}{0.24 + [0.24^2 + (0.49 + 0.5)^2]^{1/2}}\right] \\ &= 431.68\ \text{N}\cdot\text{m} \end{aligned}$$

$$\frac{T_{max}}{T_{FL}} = \frac{431.68}{163.11} = 2.65$$

(d)

$$s_{T_{max}} = \frac{R_2' + R_{ext}'}{[0.24^2 + (0.49 + 0.5)^2]^{1/2}} = \frac{R_2' + R_{ext}'}{1.0186}$$

$$R_{ext}' = 1.0186 - 0.2 = 0.8186\ \Omega/\text{phase}$$

Note that for parts (a) and (b), it is not necessary to use Thevenin's equivalent circuit. Calculation can be based on the equivalent circuit of Fig. 5.15 as follows:

$$\begin{aligned} Z_1 &= R_1 + jX_1 + R_e + jX_e \\ &= 0.25 + j0.5 + 5.598 + j1.596 \\ T &= \frac{3}{\omega_{syn}} I_1^2 R_e \\ &= \frac{3}{188.5} \times 42.754^2 \times 5.598 \\ &= 163\ \text{N}\cdot\text{m} \quad \blacksquare \end{aligned}$$

EXAMPLE 5.5

A three-phase, 460 V, 60 Hz, six-pole wound-rotor induction motor drives a constant load of $100\ \mathrm{N \cdot m}$ at a speed of 1140 rpm when the rotor terminals are short-circuited. It is required to reduce the speed of the motor to 1000 rpm by inserting resistances in the rotor circuit. Determine the value of the resistance if the rotor winding resistance per phase is 0.2 ohms. Neglect rotational losses. The stator-to-rotor turns ratio is unity.

Solution

The synchronous speed is

$$n_s = \frac{120 \times 60}{6} = 1200\ \mathrm{rpm}$$

Slip at 1140 rpm:

$$s_1 = \frac{1200 - 1140}{1200} = 0.05$$

Slip at 1000 rpm:

$$s_2 = \frac{1200 - 1000}{1200} = 0.167$$

From the equivalent circuits, it is obvious that if the value of R_2'/s remains the same, the rotor current I_2 and the stator current I_1 will remain the same, and the machine will develop the same torque (Eq. 5.54). Also, if the rotational losses are neglected, the developed torque is the same as the load torque. Therefore, for unity turns ratio,

$$\frac{R_2}{s_1} = \frac{R_2 + R_{\mathrm{ext}}}{s_2}$$

$$\frac{0.2}{0.05} = \frac{0.2 + R_{\mathrm{ext}}}{0.167}$$

$$R_{\mathrm{ext}} = 0.468\ \Omega/\mathrm{phase} \quad \blacksquare$$

EXAMPLE 5.6

The rotor current at start of a three-phase, 460 volt, 1710 rpm, 60 Hz, four-pole, squirrel-cage induction motor is six times the rotor current at full load.

(a) Determine the starting torque as percent of full-load torque.
(b) Determine the slip and speed at which the motor develops maximum torque.
(c) Determine the maximum torque developed by the motor as percent of full-load torque.

Solution

Note that the equivalent circuit parameters are not given. Therefore, the equivalent circuit parameters cannot be used directly for computation.

(a) The synchronous speed is

$$n_s = \frac{120 \times 60}{4} = 1800\,\text{rpm}$$

The full-load slip is

$$s_{FL} = \frac{1800 - 1710}{1800} = 0.05$$

From Eq. 5.52a,

$$T = \frac{I_2^2 R_2}{\omega_{syn}} \propto \frac{I_2^2 R_2}{s}$$

Thus,

$$\frac{T_{st}}{T_{FL}} = \left| \frac{I_{2(st)}}{I_{2(FL)}} \right|^2 s_{FL}$$

$$\begin{aligned} T_{st} &= 6^2 \times 0.05 \times T_{FL} = 1.8 T_{FL} \\ &= 180\%\, T_{FL} \end{aligned}$$

(b) From Eq. 5.64,

$$\frac{T_{st}}{T_{max}} = \frac{2 s_{T_{max}}}{1 + s_{T_{max}}^2}$$

$$\frac{T_{FL}}{T_{max}} = \frac{2 s_{T_{max}} s_{FL}}{s_{T_{max}}^2 + s_{FL}^2}$$

From these two expressions,

$$\frac{T_{st}}{T_{FL}} = \frac{s_{T_{max}}^2 + s_{FL}^2}{s_{FL} + s_{FL} \times s_{T_{max}}^2}$$

$$1.8 = \frac{s_{T_{max}}^2 + 0.0025}{0.05 + 0.05 \times s_{T_{max}}^2}$$

$$s_{T_{max}}^2 + 0.0025 = 0.09 + 0.09 s_{T_{max}}^2$$

$$s_{T_{max}} = \left(\frac{0.0875}{0.91} \right)^{1/2} = 0.31$$

$$\begin{aligned} \text{Speed at maximum torque} &= (1 - 0.31) \times 1800 \\ &= 1242\,\text{rpm} \end{aligned}$$

(c) From Eq. 5.64,

$$T_{\max} = \left|\frac{1 + s_{T_{\max}}^2}{2s_{T_{\max}}}\right| T_{st}$$

$$= \frac{1 + 0.31^2}{2 \times 0.31} \times 1.8T_{FL}$$

$$= 3.18T_{FL} = 318\%\, T_{FL} \quad \blacksquare$$

EXAMPLE 5.7

A three-phase source of variable frequency is required for an experiment. The frequency-changer system is as shown in Fig. E5.7. The induction machine is a three-phase, six-pole, wound-rotor type whose stator terminals are connected to a three-phase, 460 volt, 60 Hz supply. The variable-frequency output is obtained from the rotor terminals. The frequency is to be controlled over the range 15–120 Hz.

(a) Determine the speed in rpm of the system to give 15 Hz and 120 Hz.

(b) If the open-circuit rotor voltage is 240 volts when the rotor is at standstill, determine the rotor voltage available on open circuit with 15 Hz and 120 Hz.

(c) If all the losses in the machine are neglected, what fraction of the output power is supplied by the ac supply, and what fraction is supplied by the dc machine at 15 Hz and 120 Hz?

Solution

(a) For $f_2 = 15$ Hz, the slip is

$$s = \pm\frac{f_2}{f_1} = \pm\frac{15}{60} = \pm\frac{1}{4}$$

The synchronous speed is

$$n_s = \frac{120 \times 60}{6} = 1200 \text{ rpm}$$

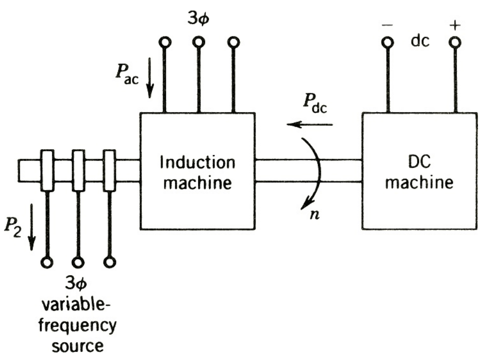


FIGURE E5.7 Induction machine as phase shifter.

The speed of the system for $f_2 = 15$ Hz is

$$n = (1 \pm s)n_s = \left(1 \pm \frac{1}{4}\right) \times 1200$$

$$= 900 \text{ and } 1500 \text{ rpm}$$

For $f_2 = 120$ Hz,

$$s = \pm\frac{120}{60} = \pm 2.0$$

$$n = (1 \pm 2.0)1200$$

$$= -1200 \text{ and } 3600 \text{ rpm}$$

(b)

$$sE_2 = s \times 240$$

For $f_2 = 15$ Hz, $sE_2 = 60$ V
For $f_2 = 120$ Hz, $sE_2 = 480$ V

(c) Power from the supply:

$$P_{ac} = P_{ag} = \frac{P_2}{s}$$

Power from the shaft:

$$P_{dc} = -(1 - s)P_{ag} = -\frac{(1 - s)}{s} \times P_2$$

For $f_2 = 15$ Hz,

$$P_{ac} = \frac{P_2}{+(1/4)}, \frac{P_2}{-(1/4)} = +4P_2, -4P_2$$

$$P_{dc} = \frac{-[1 - (1/4)]}{+ (1/4)}P_2, \frac{-[1 + (1/4)]}{-(1/4)}P_2$$

$$= -3P_2, + 5P_2$$

For $f_2 = 120$ Hz,

$$P_{ac} = \frac{P_2}{2.0}, \frac{P_2}{- 2.0} = 0.5P_2, -0.5P_2$$

$$P_{dc} = \frac{-(1 - 2.0)}{+ 2.0}P_2, \frac{-(1 + 2.0)}{- 2.0}P_2$$

$$= 0.5P_2, 1.5P_2$$

The results are summarized in the following table:

f_2 (Hz)	rpm	Mode of Operation of Induction Machine	Slip	sE_2 (V)	Stator Input, P_{ac}	Shaft Input, P_{dc}	Rotor Output
15	900	Motor	+(1/4)	60	$4P_2$	$-3P_2$	P_2
	1500	Generator	−(1/4)	60	$-4P_2$	$5P_2$	P_2
120	−1200	Plugging	+2.0	480	$0.5P_2$	$0.5P_2$	P_2
	3600	Generator	−2.0	480	$-0.5P_2$	$1.5P_2$	P_2

For practical reasons, high-speed operation should be avoided. The speed range is therefore 900 to −1200 rpm for the varying output frequency in the range 15 to 120 Hz. The speed 900 rpm implies that the dc machine rotates in the same direction as the rotating field, whereas −1200 rpm implies that the dc machine rotates opposite to the rotating field. ■

5.11 EFFECTS OF ROTOR RESISTANCE

In a conventional squirrel-cage motor at full load, the slip and the current are low, but the power factor and the efficiency are high. However, at start, the torque and power factor are low, but the current is high. If the load requires a high starting torque (Fig. 5.24), the motor will accelerate slowly. This will make a large current flow for a longer time, thereby creating a heating problem.

The resistances in the rotor circuit greatly influence the performance of an induction motor. A low rotor resistance is required for normal operation, when running, so that the slip is low and the efficiency high. However, a higher rotor resistance is required for starting so that the starting torque and power factor are high and the starting current is low. An induction motor with a fixed rotor circuit resistance therefore requires a compromise design of the rotor for starting and running conditions. Various types of induction motors are available in which the rotor circuit resistance is changed or can change with speed to suit the particular application.

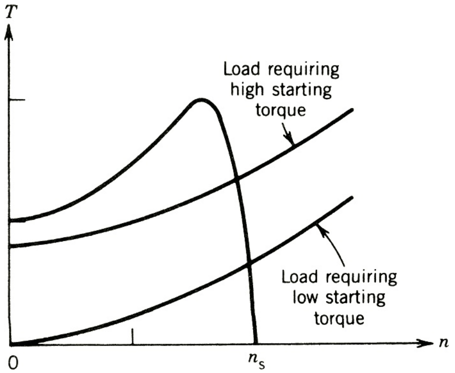


FIGURE 5.24 Loads with different torque requirements.

5.11.1 WOUND-ROTOR MOTORS

In wound-rotor induction motors external resistances can be connected to the rotor winding through slip rings (Fig. 5.39*a*). Equation 5.58 shows that the slip at which maximum torque occurs is directly proportional to the rotor circuit resistance.

$$s_{T_{\max}} \propto (R_{w2} + R_{ext}) \tag{5.71}$$

where R_{w2} is the per-phase rotor winding resistance

R_{ext} is the per-phase resistance connected externally to the rotor winding

The external resistance R_{ext} can be chosen to make the maximum torque occur at standstill $(s_{T_{\max}}) = 1$ if high starting torque is desired. This external resistance can be decreased as the motor speeds up, making maximum torque available over the whole accelerating range, as shown in Fig. 5.25. Equation 5.59 indicates that the maximum torque remains the same, as it is independent of the rotor circuit resistance.

Note that most of the rotor I^2R loss is dissipated in the external resistances. Thus, the rotor heating is lower during the starting and acceleration period than it would be if the resistances were incorporated in the rotor windings. The external resistance is eventually cut out so that under running conditions the rotor resistance is only the rotor winding resistance, which is designed to be low to make the rotor operate at high efficiency and low full-load slip.

Apart from high starting torque requirements, the external resistance can also be used for varying the running speed. This will be discussed in Section 5.13.6.

The disadvantage of the wound-rotor induction machine is its higher cost than the squirrel-cage motor.

5.11.2 DEEP-BAR SQUIRREL-CAGE MOTORS

The rotor frequency changes with speed. At standstill, the rotor frequency equals the supply frequency. As the motor speeds up, the rotor frequency decreases to a low value. At full-load running condition, the rotor frequency is in the range of 1 to 3 Hz with a 60 Hz supply connected to the stator terminals. This fact can be utilized to change the rotor resistance

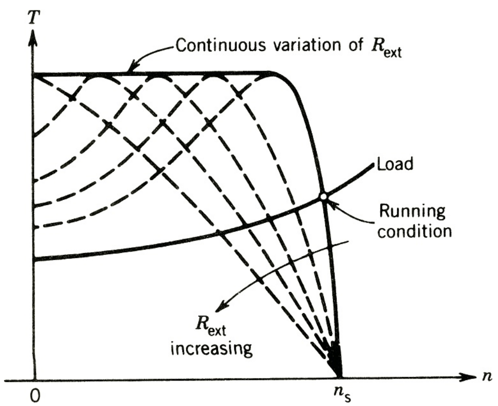


FIGURE 5.25 Maximum torque obtained by varying rotor resistance throughout the speed range.

automatically. The rotor bars can be properly shaped and arranged so that their effective resistance at supply frequency (say 60 Hz) is several times the resistance at full-load rotor frequency (1 to 3 Hz). This change in the resistance of the rotor bars is due to what is commonly known as the skin effect.

Figure 5.26*a* shows a squirrel-cage rotor with deep and narrow bars. The slot leakage fluxes produced by the current in the bar are also shown in the figure. All the leakage flux lines will close paths below the slot. It is obvious that the leakage inductance of the bottom layer is higher than that of the top layer because the bottom layer is linked with more leakage flux. The current in the high-reactance bottom layer will be less than the current in the low-reactance upper layer; that is, the rotor current will be pushed to the top of the bars. The result will be an increase in the effective resistance of the bar. Because this nonuniform current distribution depends on the reactance, it is more pronounced at high frequency (i.e., when the motor is at standstill) than at low frequency (i.e., when the motor is running at full speed). The deep rotor bars may be designed so that the effective rotor resistance at standstill is several times its effective resistance at rated speed. At full speed, the rotor frequency is very low (1–3 Hz), the rotor current is almost uniformly distributed over the cross section of the rotor bar, and the rotor ac effective resistance, R_{ac}, is almost the same as the dc resistance, R_{dc}. A typical variation of the ratio R_{ac}/R_{dc} with rotor frequency is shown in Fig. 5.26*b*.

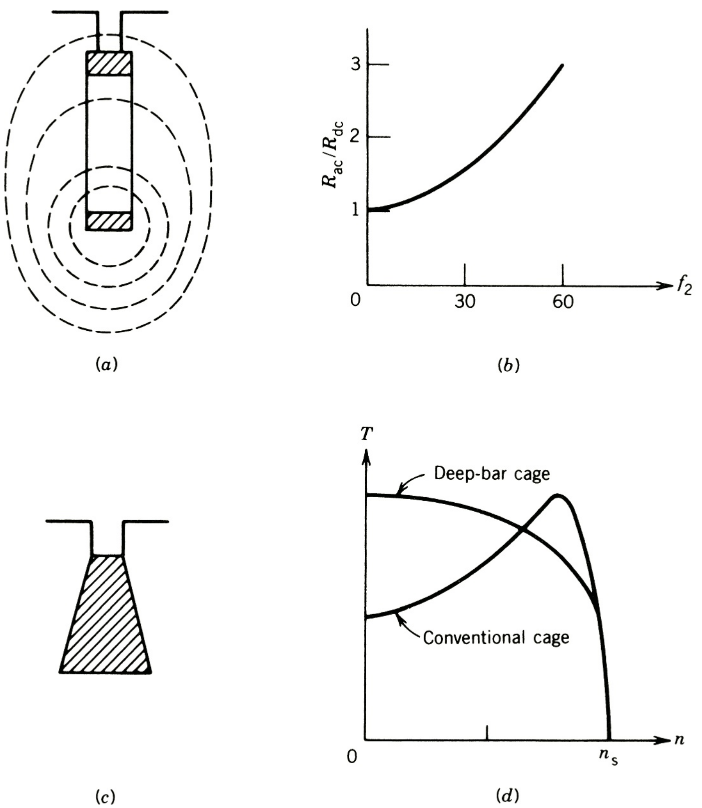


FIGURE 5.26 Deep-bar rotor and characteristics.

The rotor bars can take other shapes, such as broader at the base than at the top of the slot, as shown in Fig. 5.26*c*. This shape will further increase the ratio R_{ac}/R_{dc} with frequency.

With proper design of the rotor bars, a flat-topped torque–speed characteristic, as shown in Fig. 5.26*d*, can be obtained.

5.11.3 DOUBLE-CAGE ROTORS

Low starting current and high starting torque may also be obtained by building a double-cage rotor. In this type of rotor construction, the squirrel-cage windings consist of two layers of bars (as shown in Fig. 5.27*a*), each layer short-circuited by end rings. The outer cage bars have a smaller cross-sectional area than the inner bars and are made of relatively higher-resistivity material. Thus, the resistance of the outer cage is greater than the resistance of the inner cage. The leakage inductance of the inner cage is increased by narrowing the slots above it. At standstill most of the rotor current flows through the upper cage, thereby increasing the effective resistance of the rotor circuit. At low rotor frequencies, corresponding to low slips (such as at full-load running condition), reactance is negligible, the lower-cage bars share the current with the outer bars, and the rotor resistance approaches that of two cages in parallel. Because both cages carry current under normal running conditions, the rating of the motor is somewhat increased.

In both double-cage and deep-bar rotors, the effective resistance and leakage inductance vary with the rotor frequency. The equivalent circuit of a double-cage rotor can be represented by the circuit shown in Fig. 5.27*b*, where

L'_2 = per-phase leakage inductance of the upper-cage bars
R'_2 = per-phase resistance of the upper-cage bars
L''_2 = per-phase leakage inductance of the lower-cage bars
R''_2 = per-phase resistance of the lower-cage bars

It should be recognized that the values of these parameters depend on the rotor frequency.

Both double-cage and deep-bar cage rotors can be designed to provide the good starting characteristics resulting from higher rotor resistance and, concurrently, the good running characteristics resulting from low rotor resistance. The rotor design in these cases is based on a

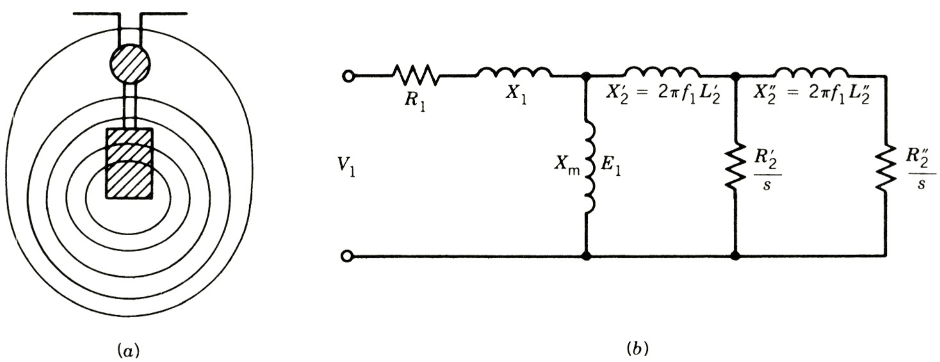


FIGURE 5.27 Double-cage rotor bars and the equivalent circuit.

compromise between starting and running performance. However, these motors are not as flexible as the wound-rotor machine with external rotor resistance. In fact, when starting requirements are very severe, the wound-rotor motor should be used.

5.12 CLASSES OF SQUIRREL-CAGE MOTORS

Industrial needs are diverse. To meet the various starting and running requirements of a variety of industrial applications, several standard designs of squirrel-cage motors are available from manufacturers' stock. The torque–speed characteristics of the most common designs, readily available and standardized in accordance with the criteria established by the National Electrical Manufacturers' Association (NEMA), are shown in Fig. 5.28. The most significant design variable in these motors is the effective resistance of the rotor cage circuits.

Class A Motors

Class A motors are characterized by normal starting torque, high starting current, and low operating slip. The motors have low rotor circuit resistance and therefore operate efficiently with a low slip ($0.005 < s < 0.015$) at full load. These machines are suitable for applications where the load torque is low at start (such as fan or pump loads) so that full speed is achieved rapidly, thereby eliminating the problem of overheating during starting. In larger machines, low-voltage starting is required to limit the starting current.

Class B Motors

Class B motors are characterized by normal starting torque, low starting current, and low operating slip. The starting torque is almost the same as that in class A motors, but the starting current is about 75 percent of that for class A. The starting current is reduced by designing for relatively high leakage reactance by using either deep-bar rotors or double-cage rotors. The high leakage reactance lowers the maximum torque. The full-load slip and efficiency are as good as those of class A motors.

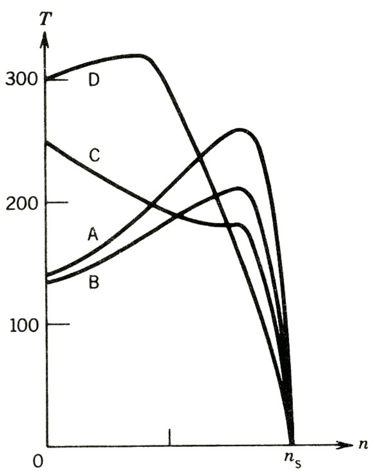


FIGURE 5.28 Torque–speed characteristics for different classes of induction motors.

Motors of this class are good general-purpose motors and have a wide variety of industrial applications. They are particularly suitable for constant-speed drives, where the demand for starting torque is not severe. Examples are drives for fans, pumps, blowers, and motor–generator sets.

Class C Motors

Class C motors are characterized by high starting torque and low starting current. A double-cage rotor is used with higher rotor resistance than is found in class B motors. The full-load slip is somewhat higher and the efficiency lower than for class A and class B motors. Class C motors are suitable for driving compressors, conveyors, crushers, and so forth.

Class D Motors

Class D motors are characterized by high starting torque, low starting current, and high operating slip. The rotor cage bars are made of high-resistance material such as brass instead of copper. The torque–speed characteristic is similar to that of a wound-rotor motor with some external resistance connected to the rotor circuit. The maximum torque occurs at a slip of 0.5 or higher. The full-load operating slip is high (8 to 15 percent), and therefore the running efficiency is low. The high losses in the rotor circuit require that the machine be large (and hence expensive) for a given power. These motors are suitable for driving intermittent loads requiring rapid acceleration and high-impact loads such as punch presses or shears. In the case of impact loads, a flywheel is fitted to the system. As the motor speed falls appreciably with load impact, the flywheel delivers some of its kinetic energy during the impact.

5.13 SPEED CONTROL

An induction motor is essentially a constant-speed motor when connected to a constant-voltage and constant-frequency power supply. The operating speed is very close to the synchronous speed. If the load torque increases, the speed drops by a very small amount. It is therefore suitable for use in substantially constant-speed drive systems. Many industrial applications, however, require several speeds or a continuously adjustable range of speeds. Traditionally, dc motors have been used in such adjustable-speed drive systems. However, dc motors are expensive, require frequent maintenance of commutators and brushes, and are prohibitive in hazardous atmospheres. Squirrel-cage induction motors, on the other hand, are cheap and rugged have no commutators, and are suitable for high-speed applications. The availability of solid-state controllers, although more complex than those used for dc motors, has made it possible to use induction motors in variable-speed drive systems.

In this section various methods for controlling the speed of an induction motor are discussed.

5.13.1 POLE CHANGING

Because the operating speed is close to the synchronous speed, the speed of an induction motor can be changed by changing the number of poles of the machine (Eq. 5.10). This can be done by changing the coil connections of the stator winding. Normally, poles are changed in the ratio 2 to 1. This method provides two synchronous speeds. If two independent sets of polyphase windings are used, each arranged for pole changing, four synchronous speeds can be obtained for the induction motor.

Squirrel-cage motors are invariably used in this scheme, because the rotor can operate with any number of stator poles. It is obvious, however, that the speed can be changed only in discrete steps, and that the elaborate stator winding makes the motor expensive.

5.13.2 LINE VOLTAGE CONTROL

Recall that the torque developed in an induction motor is proportional to the square of the terminal voltage. A set of $T-n$ characteristics with various terminal voltages is shown in Fig. 5.29. If the rotor drives a fan load, the speed can be varied over the range n_1 to n_2 by changing the line voltage. Note that the class D motor will allow speed variation over a wider speed range.

The terminal voltage V_1 can be varied by using a three-phase autotransformer or a solid-state voltage controller, as shown in Fig. 5.30. The auto-transformer provides a sinusoidal voltage for the induction motor, whereas the motor terminal voltage with a solid-state controller is non-sinusoidal. Speed control with a solid-state controller is commonly used with small squirrel-cage motors driving fan loads. In large power applications, an input filter is required; otherwise, large harmonic currents will flow in the supply line.

The thyristor voltage controller shown in Fig. 5.30*b* is simple to understand, but complicated to analyze. The operation of the voltage controller is discussed in Chapter 10. The command signal for a particular set speed fires the thyristors at a particular firing angle (α) to provide a particular terminal voltage for the motor. If the speed command signal is changed, the firing angle (α) of the thyristors changes, which results in a new terminal voltage and thus a new operating speed.

Open-loop operation is not satisfactory if precise speed control is desired for a particular application. In such a case, closed-loop operation is needed. Figure 5.30*c* shows a simple block diagram of a drive system with closed-loop operation. If the motor speed falls because of any disturbance, such as supply voltage fluctuation, the difference between the set speed and the

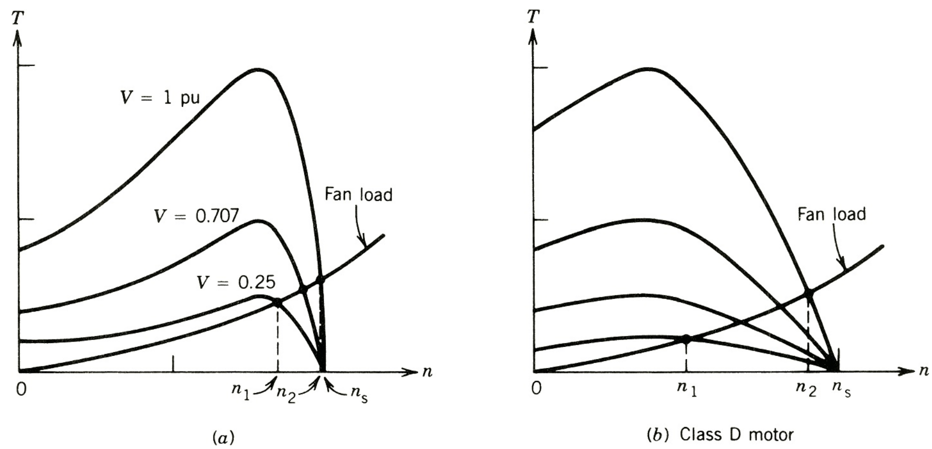


FIGURE 5.29 Torque–speed characteristics for various terminal voltages.

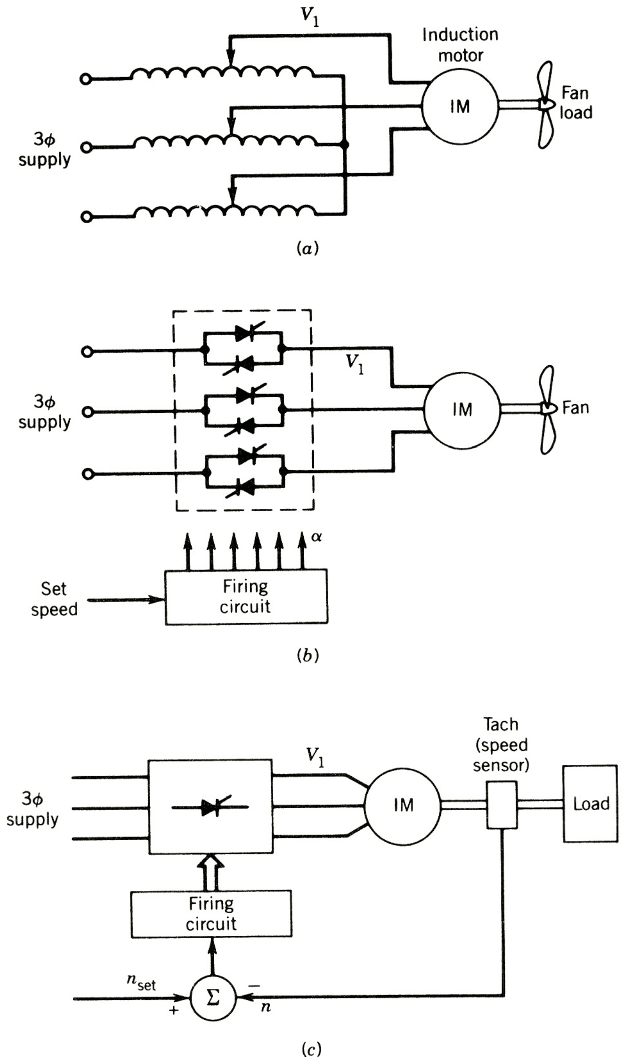


FIGURE 5.30 Starting and speed control. (*a*) Autotransformer voltage controller. (*b*) Solid-state voltage controller. (*c*) Closed-loop operation of voltage controller.

motor speed increases. This changes the firing angle of the thyristors to increase the terminal voltage, which in turn develops more torque. The increased torque tends to restore the speed to the value prior to the disturbance.

Note that for this method of speed control the slip increases at lower speeds (Fig. 5.29), making the operation inefficient. However, for fans, or similar centrifugal loads in which torque varies approximately as the square of the speed, the power decreases significantly with decrease in speed. Therefore, although the power lost in the rotor circuit ($=sP_{ag}$) may be a significant portion of the air gap power, the air gap power itself is small, and therefore the rotor will not overheat. The voltage controller circuits are simple and, although inefficient, are suitable for fan, pump, and similar centrifugal drives.

5.13.3 LINE FREQUENCY CONTROL

The synchronous speed and hence the motor speed can be varied by changing the frequency of the supply. Application of this speed control method requires a frequency changer. Figure 5.31 shows a block diagram of an open-loop speed control system in which the supply frequency of the induction motor can be varied. The operation of the controlled rectifier (ac to dc) and the inverter (dc to ac) is described in Chapter 10.

From Eq. 5.27, the motor flux is

$$\Phi_p \propto \frac{E}{f} \tag{5.72}$$

If the voltage drop across R_1 and X_1 (Fig. 5.15) is small compared to the terminal voltage V_1—that is, $V_1 \approx E_1$—then

$$\Phi_p \propto \frac{V}{f} \tag{5.73}$$

To avoid high saturation in the magnetic system, the terminal voltage of the motor must be varied in proportion to the frequency. This type of control is known as *constant volts per hertz*. At low frequencies, the voltage drop across R_1 and X_1 (Fig. 5.15) is comparable to the terminal voltage V_1, and therefore Eq. 5.73 is no longer valid. To maintain the same air gap flux density, the ratio V/f is increased for lower frequencies. The required variation of the supply voltage with frequency is shown in Fig. 5.32. In Fig. 5.31 the machine voltage will change if the input voltage to the inverter V_i is changed; V_i can be changed by changing the firing angle of the controlled rectifier. If the output voltage of the inverter can be changed in the inverter itself

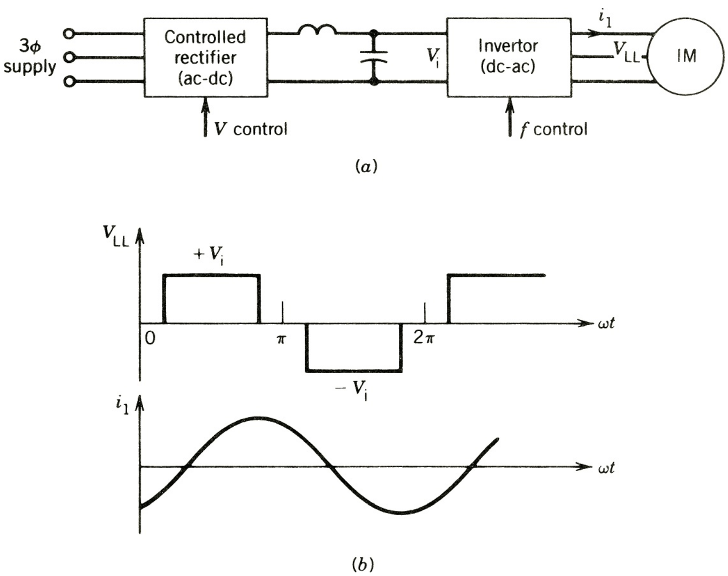


FIGURE 5.31 Open-loop speed control of an induction motor by input voltage and frequency control.

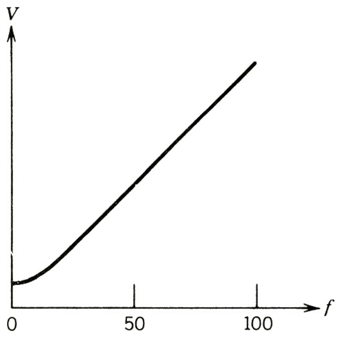


FIGURE 5.32 Required variation in voltage with change in frequency to maintain constant air gap flux density.

(as in pulse-width-modulated inverters), the controlled rectifier can be replaced by a simple diode rectifier circuit, which will make V_i constant.

The torque–speed characteristics for variable-frequency operation are shown in Fig. 5.33. At the base frequency f_{base} the machine terminal voltage is the maximum that can be obtained from the inverter. Below this frequency, the air gap flux is maintained constant by changing V_1 with f_1; hence, the same maximum torques are available. Beyond f_{base}, since V_1 cannot be further increased with frequency, the air gap flux decreases, and so does the maximum available torque. This corresponds to the field-weakening control scheme used with dc motors. Constant-horsepower operation is possible in the field-weakening region.

In Fig. 5.33 the torque–speed characteristic of a load is superimposed on the motor torque–speed characteristic. Note that the operating speeds $n_1 \cdots n_8$ are close to the corresponding synchronous speeds. In this method of speed control, therefore, the operating slip is low and efficiency is high.

The inverter in Fig. 5.31 is known as a voltage source inverter. The motor line-to-line terminal voltage is a quasi-square wave of 120° width. However, because of motor inductance, the motor current is essentially sinusoidal. A current source inverter (see Chapter 10) can be used

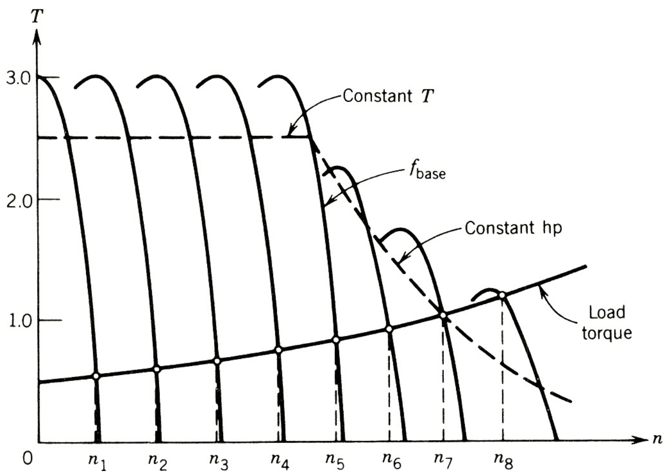


FIGURE 5.33 Torque–speed characteristics of an induction motor with variable-voltage, variable-frequency control.

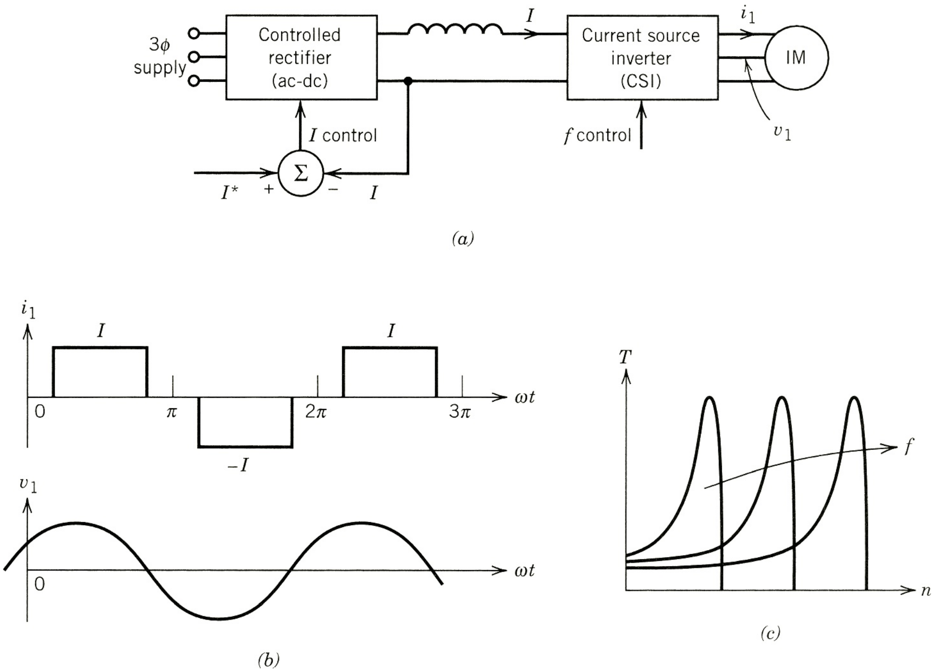


FIGURE 5.34 Induction motor drive system with a current source inverter and the corresponding characteristics.

to control the speed of an induction motor. The open-loop block diagram of a drive system using a current source inverter is shown in Fig. 5.34. The magnitude of the current is regulated by providing a current loop around the rectifier as shown in Fig. 5.34*a*. The filter inductor in the dc link smooths out the current. The motor current waveform is a quasi-square wave having 120° pulse width. The motor terminal voltage is essentially sinusoidal. The torque–speed characteristics of an induction motor fed from a current source inverter are also shown in Fig. 5.34. These characteristics have a very steep slope near synchronous speed. Although a current source inverter is rugged and desirable from the standpoint of protection of solid-state devices, the drive system should be properly operated, otherwise the system will not be stable.

5.13.4 CONSTANT-SLIP FREQUENCY OPERATION

For efficient operation of an induction machine, it is desirable to operate it at a fixed or controlled slip frequency (which is also the rotor circuit frequency). High efficiency and high power factors are obtained if the slip frequency f_2 is maintained below the breakdown frequency f_{2b}, which is the rotor circuit frequency at which the maximum torque is developed.

Consider the block diagram of Fig. 5.35. The signal f_n represents a frequency corresponding to the speed of the motor. To this a signal f_2 representing the slip (or rotor circuit) frequency is added or subtracted. The resultant f_1 represents the stator frequency:

$$f_1 = f_n \pm f_2 \tag{5.74}$$

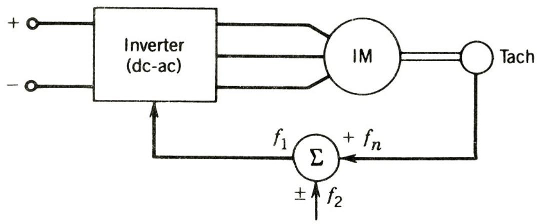


FIGURE 5.35 Constant slip frequency (f_2) operation.

Addition of f_2 to f_n will correspond to motoring action, and subtraction of f_2 from f_n will correspond to regenerative braking action of the induction machine. At any speed of the motor, the signal f_2 will represent the rotor circuit frequency—that is, the slip frequency.

5.13.5 CLOSED-LOOP CONTROL

Most industrial drive systems operate as closed-loop feedback systems. Figure 5.36 shows a block diagram of a speed control system employing slip frequency regulation and constant-volt/hertz operation. At the first summer junction the difference between the set speed n^* and the actual speed n represents the slip speed n_{sl}, and hence the slip frequency. If the slip frequency nears the breakdown frequency, its value is clamped, thereby restricting the operation below the breakdown frequency. At the second summer, the slip frequency is added to the frequency f_n (representing the motor speed) to generate the stator frequency f_1. The function generator provides a signal such that a voltage is obtained from the controlled rectifier for *constant-volt/hertz* operation.

A simple speed control system using a current source inverter is shown in Fig. 5.37. The slip frequency is kept constant, and the speed is controlled by controlling the dc link current I_d, and hence the magnitude of the motor current.

In traction applications, such as subway cars or other transit vehicles, the torque is controlled directly. A typical control scheme for transit drive systems is shown in Fig. 5.38. As the voltage available for a transit system is a fixed-voltage dc supply, a pulse-width-modulated

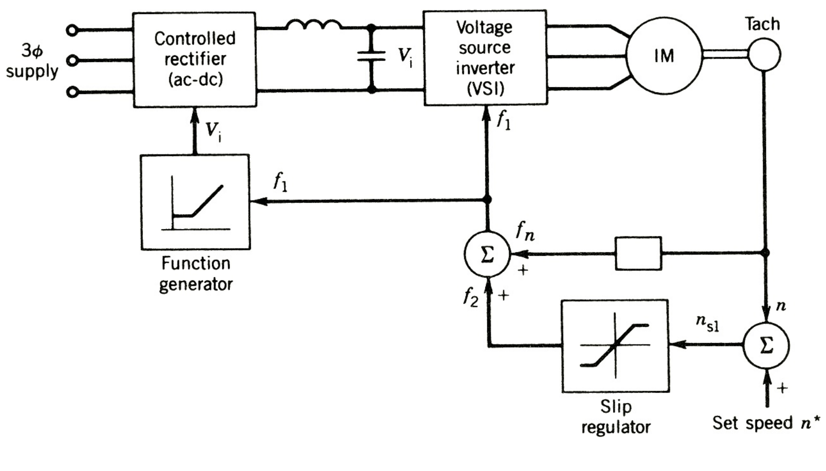


FIGURE 5.36 Speed control system employing slip frequency regulation and constant V/f operation.

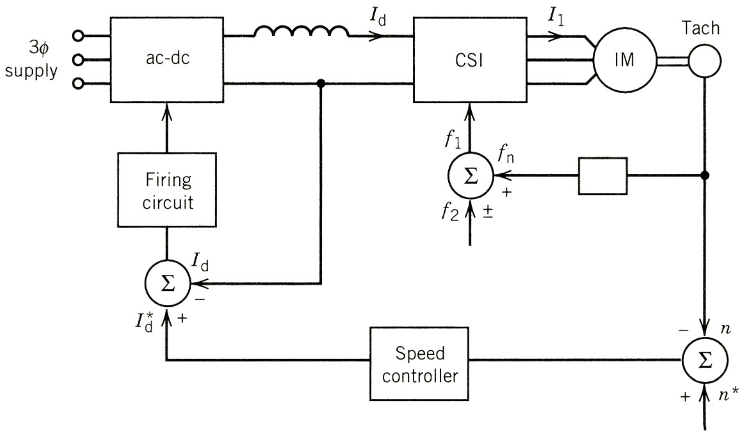


FIGURE 5.37 Speed control system with constant slip frequency (using a current source inverter).

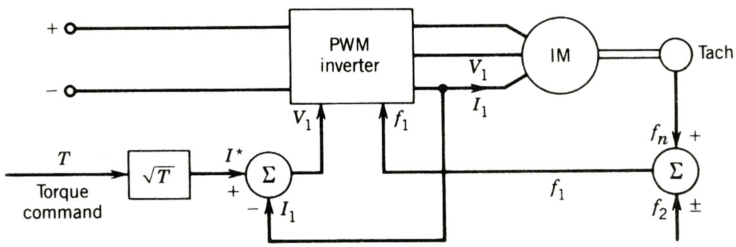


FIGURE 5.38 Typical speed control scheme for transit drive systems.

(PWM) voltage source inverter is considered in which the output voltage can be varied. It can be shown (Example 5.8) that if the slip frequency is kept constant, the torque varies as the square of the stator current. The torque command is fed through a square root function generator to produce the current reference I^*. The signal representing the difference between I^* and the actual current I_1 will change the output voltage of the PWM inverter to make I_1 close to the desired value of I^* representing the torque command. For regenerative braking of the transit vehicle, the sign of the slip frequency f_2 is negative. The induction motor will operate in the generating mode ($f_n > f_1$) and feed back the kinetic energy stored in the drive system to the dc supply.

EXAMPLE 5.8

Show that if the slip frequency is kept constant, the torque developed by an induction machine is proportional to the square of the input current.

Solution

For variable-frequency operation, the terminal voltage V_1 is changed with frequency f_1 to maintain machine flux at a desired level. In the low-frequency region, V_1 is low. The voltage drop across R_1 and X_1 may be comparable to V_1. Therefore, V_1 cannot be assumed to be equal to E_1. In the equivalent circuit, the shunt branch X_m should not be moved to the machine terminals. Therefore, for variable-frequency operation, the equivalent circuit of Fig. 5.15 is more appropriate to use for prediction of performance. From Fig. 5.15,

$$I'_2 = \frac{jX_{\mathrm{m}}}{(R'_2/s) + j(X'_2 + X_{\mathrm{m}})} I_1 \tag{5.75}$$

$$(I'_2)^2 = \frac{(X_{\mathrm{m}})^2}{(R'_2/s)^2 + (X'_2 + X_{\mathrm{m}})^2} I_1{}^2 \tag{5.76}$$

$$T = \frac{1}{\omega_{\mathrm{syn}}} I'^2_2 \frac{R^2}{s} \tag{5.77}$$

where

$$\omega_{\mathrm{syn}} = \frac{4\pi}{p} f_1 \quad \text{and} \quad s = \frac{f_2}{f_1}$$

From Eqs. 5.76 and 5.77,

$$T = \frac{\pi p L_{\mathrm{m}}^2}{R'_2} I_1^2 \frac{f_2}{1 + \left|\dfrac{2\pi(L_{\mathrm{m}} + L'_2) f_2}{R'_2}\right|^2} \tag{5.78}$$

Equation 5.78 shows that if f_2 remains constant,

$$T \propto I_1^2 \quad \blacksquare \tag{5.79}$$

EXAMPLE 5.9

Show that if the rotor frequency f_2 is kept constant, the torque developed by an induction machine is proportional to the square of the flux in the air gap.

Solution

From the equivalent circuit of Fig. 5.15,

$$I_1^2 = \frac{E_1}{\left|(R'_2 f_1/f_2)^2 + (2\pi f_1 L'_2)^2\right|^{1/2}} \tag{5.80}$$

From Eqs. 5.77 and 5.80,

$$T = \frac{p}{4\pi R'_2} \left(\frac{E_1}{f_1}\right)^2 \frac{f_2}{1 + (2\pi f_2 L'_2/R'_2)^2} \tag{5.81}$$

If f_2 remains constant, from Eqs. 5.81 and 5.72,

$$T \propto (E_1/f_1)^2 \propto \Phi_{\mathrm{p}}^2 \quad \blacksquare \tag{5.82}$$

5.13.6 CONSTANT-FLUX, Φ_p (OR E/f) OPERATION

To achieve high torque throughout the speed range, the motor air gap flux (Φ_p) should be maintained constant. The motor flux should not be allowed to decrease at lower frequencies as a result of the increasing effects of the stator resistance. As the motor flux is proportional to E/f, it can be maintained constant if the air gap voltage E, rather than terminal voltage V, is varied linearly with the stator frequency.

It is evident from Eq. 5.81 that the motor torque depends on the motor flux ($\Phi_p \propto E/f$) and rotor frequency (f_2). Under constant-flux operation, for maximum torque

$$\frac{dT}{df_2} = 0 \tag{5.83}$$

From Eqs. 5.81 and 5.83,

$$2\pi f_{2b} L_2' = R_2' \tag{5.84}$$

where f_{2b} = rotor frequency at which maximum torque is developed (also called breakdown frequency).

From Eq. 5.84,

$$f_{2b} = \frac{R_2'}{2\pi L_2'} \tag{5.85}$$

From Eqs. 5.81 and 5.85, the maximum torque per phase is

$$T_{\max} = \frac{p}{4\pi}\left(\frac{E_1}{f_1}\right)^2 \frac{1}{4\pi L_2'} \tag{5.86}$$

5.13.7 CONSTANT-CURRENT OPERATION

The motor can be operated at constant current by providing a current loop around the ac–dc converter as shown in Fig. 5.34*a*. Since the motor rms current I_1 is proportional to the dc link current I, the motor current can be maintained at a value corresponding to a set current I^*.

From Eq. 5.78, the torque developed depends on the motor current I_1 and rotor frequency f_2. Under constant-current operation, for maximum torque,

$$\frac{dT}{df_2} = 0 \tag{5.87}$$

From Eqs. 5.78 and 5.87, the condition for maximum torque is

$$2\pi(L_m + L_2')f_{2b} = R_2' \tag{5.88}$$

where f_{2b} = rotor frequency at which maximum torque is developed.

From Eq. 5.88,

$$f_{2b} = \frac{R_2'}{2\pi(L_m + L_2')} \tag{5.89}$$

From Eqs. 5.89 and 5.78, the maximum torque developed per phase is

$$
\begin{aligned}
T_{\max} &= \frac{\pi p L_m^2}{R_2'} I_1^2 \frac{f_{2b}}{2} \\
&= \frac{\pi p L_m^2}{R_2'} I_1^2 \frac{R_2'}{4\pi(L_m + L_2')} \\
&= \frac{p L_m^2}{4(L_m + L_2')} I_1^2
\end{aligned}
\tag{5.90}
$$

Equations 5.85 and 5.89 reveal that the rotor frequency at which maximum torque is developed is much smaller in the constant-current operation. For this reason the slope of the torque–speed characteristics in Fig. 5.34*c* is very steep compared with that in Fig. 5.33.

EXAMPLE 5.10

For the induction machine of Example 5.4, determine the breakdown frequencies for the following operations.

(a) Motor operated from a 3ϕ, 460 V, 60 Hz supply.
(b) Motor operated under constant-flux condition.
(c) Motor operated under constant-current condition.

Solution

(a) From Eq. 5.58,

$$
\begin{aligned}
s_{T\max} &= \frac{0.2}{\{0.24^2 + (0.49 + 0.5)^2\}^{1/2}} \\
&= 0.1963 \\
f_{2b} &= 0.1963 \times 60 \\
&= 11.7797\ \text{Hz}
\end{aligned}
$$

(b)

$$L_2' = \frac{0.5}{2\pi 60} = 1.33 \times 10^{-3}\ \text{H}$$

From Eq. 5.85,

$$f_{2b} = \frac{0.2}{2\pi\, 1.33 \times 10^{-3}} = 23.93\ \text{Hz}$$

(c)

$$L_m = \frac{30}{2\pi 60} = 79.58 \times 10^{-3}\ \text{H}$$

From Eq. 5.89,

$$f_{2b} = \frac{0.2}{2\pi(79.58 + 1.33) \times 10^{-3}}$$

$$= 0.3934\ \text{Hz} \quad \blacksquare$$

5.13.8 ROTOR RESISTANCE CONTROL

In Section 5.11.1 it was pointed out that the speed of a wound-rotor induction machine can be controlled by connecting external resistance in the rotor circuit through slip rings, as shown in Fig. 5.39*a*. The torque–speed characteristics for four external resistances are shown in Fig. 5.39*b*. The load $T{-}n$ characteristic is also shown by the dashed line. By varying the external resistance $0 < R_{ex} < R_{ex4}$, the speed of the load can be controlled in the range $n_1 < n < n_5$. Note that by proper adjustment of the external resistance ($R_{ex} = R_{ex2}$), maximum starting torque can be obtained for the load.

The scheme shown in Fig. 5.39*a* requires a three-phase resistance bank, and for balanced operation all three resistances should be equal for any setting of the resistances. Manual adjustment of the resistances may not be satisfactory for some applications, particularly for a closed-loop feedback control system. Solid-state control of the external resistance may provide smoother operation. A block diagram of a solid-state control scheme with open-loop operation is shown in Fig. 5.39*c*. The three-phase rotor power is rectified by a diode bridge. The effective value R^*_{ex} of the external resistance R_{ex} can be changed by varying the on-time (also called the duty ratio α, Chapter 10) of the chopper connected across R_{ex}. It can be shown that[1]

$$R^*_{ex} = (1 - \alpha)R_{ex} \tag{5.91}$$

When $\alpha = 0.0$, that is, when the chopper is off all the time, $R^*_{ex} = R_{ex}$. When $\alpha = 1.0$—that is, the chopper is on all the time—R_{ex} is short-circuited by the chopper, so $R^*_{ex} = 0$. In this case, the rotor circuit resistance consists of the rotor winding resistance only. Therefore, by varying α in the range $1 > \alpha > 0$, the effective resistance is varied in the range $0 < R^*_{ex} < R_{ex}$, and torque–speed characteristics similar to those shown in Fig. 5.39*b* are obtained.

The rectified voltage V_d (Fig. 5.39*c*) depends on the speed, and hence the slip, of the machine. At standstill, let the induced voltage in the rotor winding be E_2 (Eq. 5.29). From Eq. 5.34 and Eq. 10.9 (for a 3ϕ full-wave diode rectifier with six diodes), the rectified voltage V at slip s is

$$V_d = s|V_d|_{s=1} = s\frac{3\sqrt{6}}{\pi}E_2 \tag{5.92}$$

The electrical power in the rotor circuit is

$$P_2 = sP_{ag}$$

[1] P. C. Sen and K. H. J. Ma, Rotor Chopper Control for Induction Motor Drives: TRC Strategy, *IA-IEEE Transactions*, vol. IA-11, no. 1, pp. 43–49, 1975.

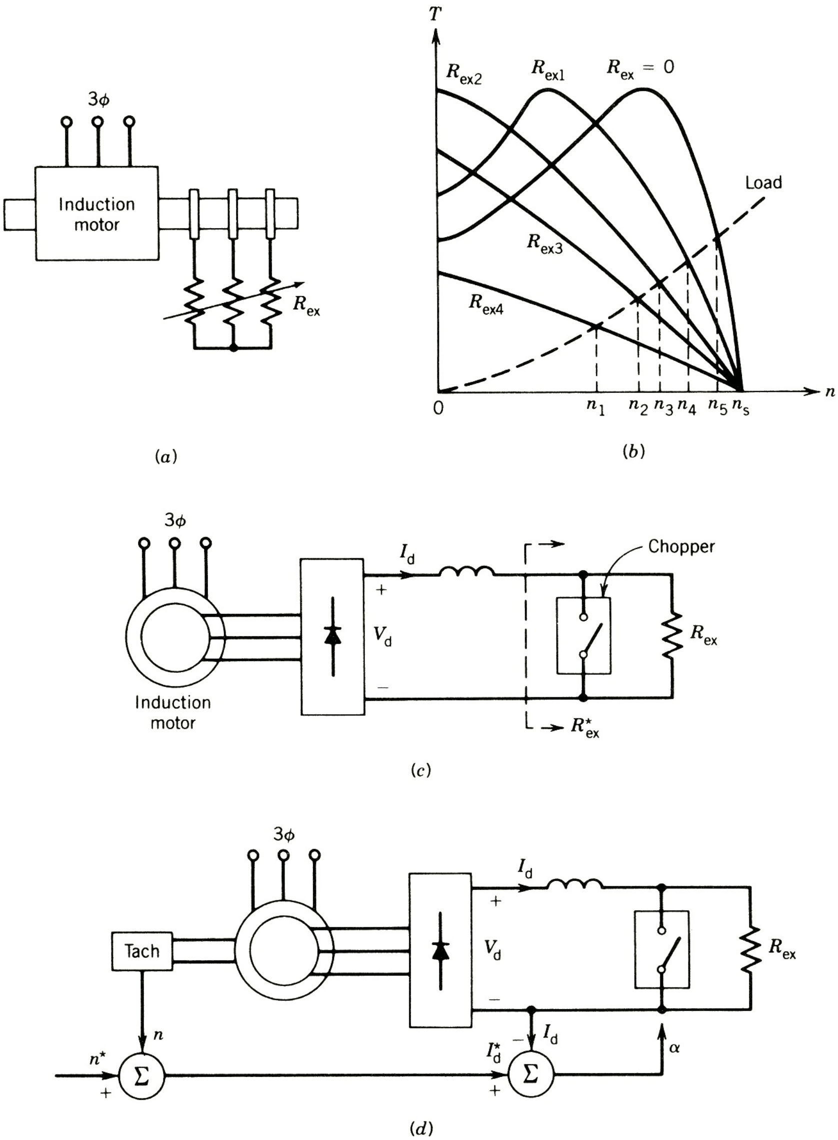


FIGURE 5.39 Speed control with rotor resistance control: open-loop and closed-loop schemes.

If the power lost in the rotor winding is neglected, the power P_2 is the dc output power of the rectifier. Hence,

$$sP_{ag} \simeq V_d I_d \tag{5.93}$$

From Eqs. 5.92 and 5.93,

$$sT\omega_{syn} = s\frac{3\sqrt{6}}{\pi}E_2 I_d \tag{5.94}$$

$$T \propto I_d$$

This linear relationship between the developed torque and the rectified current is an advantage from the standpoint of closed-loop control of this type of speed drive system. A block diagram for closed-loop operation of the solid-state rotor resistance control system is shown in Fig. 5.39*d*. The actual speed n is compared with the set speed n^*, and the error signal represents the torque command or the current reference I_d^*. This current demand I_d^* is compared with the actual current I_d, and the error signal changes the duty ratio α of the chopper to make current I_d close to the value I_d^*.

The major disadvantage of the rotor resistance control method is that the efficiency is low at reduced speed because of higher slips (Eq. 5.70). However, this control method is often employed because of its simplicity. In applications where low-speed operation is only a small proportion of the work, the low efficiency is acceptable. A typical application of the rotor resistance control method is the hoist drive of a shop crane. This method also can be used in fan or pump drives, where speed variation over a small range near the top speed is required.

5.13.9 ROTOR SLIP ENERGY RECOVERY

In the scheme just discussed, if the slip power lost in the resistance could be returned to the ac source, the overall efficiency of the drive system would be very much increased. A method for recovering the slip power is shown in Fig. 5.40*a*. The rotor power is rectified by the diode

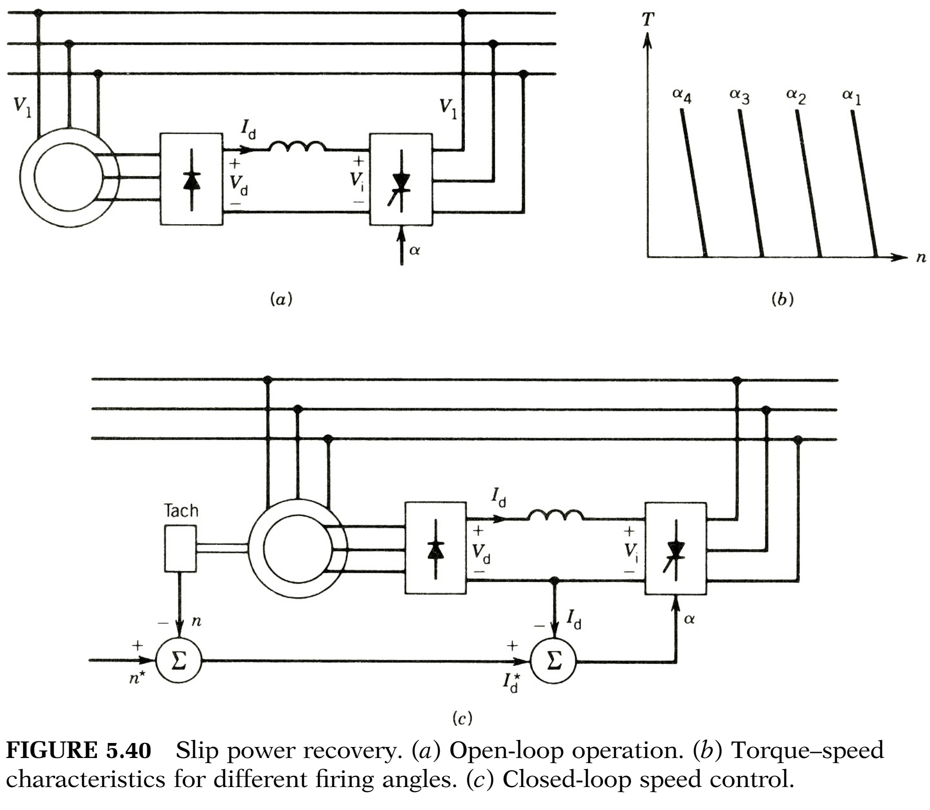


FIGURE 5.40 Slip power recovery. (*a*) Open-loop operation. (*b*) Torque–speed characteristics for different firing angles. (*c*) Closed-loop speed control.

bridge. The rectified current is smoothed out by the smoothing choke. The output of the rectifier is then connected to the dc terminals of the inverter, which inverts this dc power to ac power and feeds it back to the ac source. The inverter is a controlled rectifier operated in the inversion mode (see Chapter 10).

At no load, the torque required is very small and from Eq. 5.94 $I_d \simeq 0$.
From Fig. 5.40*a*,

$$V_{\mathrm{d}} = V_{\mathrm{i}}$$

If the no-load slip is s_0, then, from Eqs. 5.92 and 10.10 (Chapter 10),

$$s_0 \frac{3\sqrt{6}}{\pi} E_2 = \frac{-3\sqrt{6}}{\pi} V_1 \cos\alpha$$

or

$$s_0 = -(V_1/E_2)\cos\alpha \tag{5.95}$$

The firing angle α of the inverter will therefore set the no-load speed. If the load is applied, the speed will decrease. The torque-speed characteristics at various firing angles are shown in Fig. 5.40*b*. These characteristics are similar to those of a dc separately excited motor at various armature voltages.

As shown earlier, the torque developed by the machine is proportional to the dc link current I_{d} (Eq. 5.94). A closed-loop speed control system using the slip power recovery technique is shown in Fig. 5.40*c*.

This method of speed control is useful in large power applications where variation of speed over a wide range involves a large amount of slip power.

5.14 STARTING OF INDUCTION MOTORS

Squirrel-cage induction motors are frequently started by connecting them directly across the supply line. A large starting current of the order of 500 to 800 percent of full-load current may flow in the line. If this causes appreciable voltage drop in the line, it may affect other drives connected to the line. Also, if a large current flows for a long time, it may overheat the motor and damage the insulation. In such a case, reduced-voltage starting must be used.

A three-phase step-down autotransformer, as shown in Fig. 5.41*a*, may be employed as a reduced-voltage starter. As the motor approaches full speed, the autotransformer is switched out of the circuit.

A star–delta method of starting may also be employed to provide reduced voltage at start. In this method, the normal connection of the stator windings is delta while running. If these windings are connected in star at start, the phase voltage is reduced, resulting in less current at starting. As the motor approaches full speed, the windings will be connected in delta, as shown in Fig. 5.4l*b*.

A solid-state voltage controller, as shown in Fig. 5.41*c*, can also be used as a reduced-voltage starter. The controller can provide smooth starting. This arrangement can also be used to control the speed of the induction motor, as discussed earlier in Section 5.13.

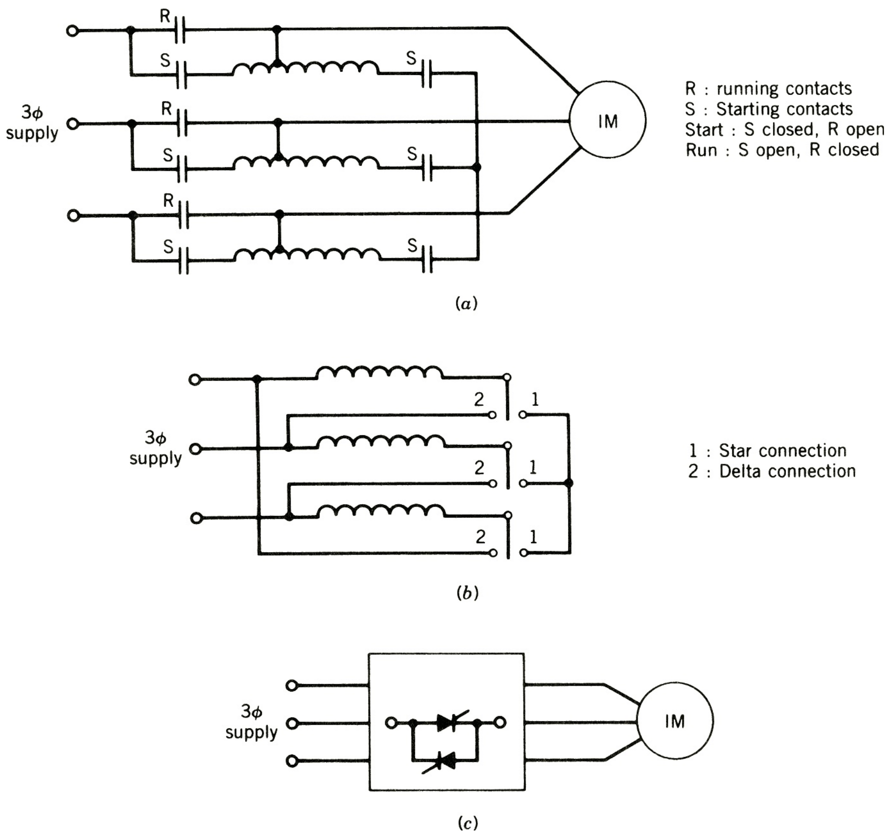


FIGURE 5.41 Starting methods for squirrel-cage induction motors. (*a*) One-step starting autotransformer. (*b*) Star–delta starting. (*c*) Solid-state voltage controller for starting.

Note that although reduced-voltage starting reduces the starting current, it also results in a decrease in the starting torque, because the torque developed is proportional to the square of the terminal voltage (see Eq. 5.54).

5.15 TIME AND SPACE HARMONICS

Induction machines are often controlled by voltage source inverters or current source inverters, as discussed in Section 5.13.3. The machine currents are therefore nonsinusoidal. They contain fundamental and harmonic components of current. The harmonic currents produce rotating fields in the air gap that rotate at higher speeds than the rotating field produced by the fundamental current. The time harmonic currents and their rotating fields produce parasitic torques in the machine.

The winding of a phase of an induction machine is distributed over a finite number of slots in the machine. As a result, when current flows through the winding the mmf produced is nonsinusoidally distributed in the air gap (see Appendix A). The air gap flux, therefore, consists of fundamental and harmonic components of fluxes. These harmonic fluxes produced by a distributed winding are known as space harmonics. The space harmonics also produce parasitic torques in the machine.

5.15.1 TIME HARMONICS

While considering the effects of time harmonics, we shall assume that when current flows through a phase winding, it produces sinusoidally distributed mmf in the air gap. In other words, we shall assume a sinusoidally distributed winding (see Appendix A) and no space harmonics.

Let phase currents in the three-phase induction machine be as follows:

$$i_{\mathrm{a}} = \sum_{h=1}^{\infty} I_{h(\max)} \cos h\omega t \tag{5.96}$$

$$i_{\mathrm{b}} = \sum_{h=1}^{\infty} I_{h(\max)} \cos h(\omega t - 120^\circ) \tag{5.97}$$

$$i_{\mathrm{c}} = \sum_{h=1}^{\infty} I_{h(\max)} \cos h(\omega t + 120^\circ) \tag{5.98}$$

where h is the order of harmonics

$I_{h(\max)}$ is the amplitude of the hth-order harmonic current

Assume that turns of a phase winding are sinusoidally distributed. From Fig. 5.6*a*, the mmf along θ due to current in phase a is

$$F_{\mathrm{a}} = N i_{\mathrm{a}} \cos\theta \tag{5.99}$$

where N is the turns per phase. From Eqs. 5.96 and 5.99,

$$F_{\mathrm{a}}(\theta, t) = \sum_{h=1}^{\infty} N I_{h(\max)} \cos h\omega t \cos\theta \tag{5.100}$$

$$= \sum_{h=1}^{\infty} F_{h(\max)} \cos h\omega t \cos\theta \tag{5.101}$$

where

$$F_{h(\max)} = N I_{h(\max)} \tag{5.102}$$

Similarly, contributions from phases b and c are, respectively,

$$F_{\mathrm{b}}(\theta, t) = \sum_{h=1}^{\infty} F_{h(\max)} \cos h(\omega t - 120^\circ) \cos(\theta - 120^\circ) \tag{5.103}$$

$$F_{\mathrm{c}}(\theta, t) = \sum_{h=1}^{\infty} F_{h(\max)} \cos h(\omega t + 120^\circ) \cos(\theta + 120^\circ) \tag{5.104}$$

The resultant mmf along θ is

$$F(\theta, t) = F_a(\theta, t) + F_b(\theta, t) + F_c(\theta, t) \tag{5.105}$$

$$= \sum_{h=1}^{\infty} F_{h(\max)}[\cos(h\omega t)\cos\theta + \cos h(\omega t - 120^\circ)\cos(\theta - 120^\circ) + \cos h(\omega t + 120^\circ)\cos h(\theta - 120^\circ)] \tag{5.106}$$

Fundamental mmf

From Eq. 5.106,

$$\begin{aligned} F_1(\theta, t) &= F_{1(\max)}[\cos\omega t\cos\theta + \cos(\omega t - 120^\circ)\cos(\theta - 120^\circ) \\ &\quad + \cos(\omega t + 120^\circ)\cos(\theta - 120^\circ)] \\ &= \frac{3}{2}F_1(\max)\cos(\theta - \omega t) \end{aligned} \tag{5.107}$$

The fundamental mmf is therefore a rotating mmf that rotates in the forward direction (i.e., in the direction of θ) at an angular speed of ω radians per second.

Third Harmonic mmf

From Eq. 5.106,

$$\begin{aligned} F_3(\theta, t) &= F_{3(\max)}\{\cos 3\omega t[\cos\theta + \cos(\theta - 120^\circ) + \cos(\theta + 120^\circ)]\} \\ &= F_{3(\max)} \times 0 \\ &= 0 \end{aligned} \tag{5.108}$$

Note that in a three-phase, three-wire system, third harmonic current is absent. Therefore, $F_{3(\max)}$ is also zero.

Fifth Harmonic mmf

From Eq. 5.106, it can be shown that

$$F_5(\theta, t) = \tfrac{3}{2}F_{5(\max)}\cos(\theta + 5\omega t) \tag{5.109}$$

The fifth harmonic mmf wave is also a rotating wave that rotates in the opposite direction (with respect to the rotation of the fundamental wave) and at five times the speed of the fundamental wave.

Seventh Harmonic mmf

From Eq. 5.106, it can be shown that

$$F_7(\theta, t) = \tfrac{3}{2}F_{7(\max)}\cos(\theta - 7\omega t) \tag{5.110}$$

TABLE 5.1 Synchronous Speeds for Different Time Harmonic mmf Waves in a 3ϕ, Four-Pole, 60 Hz Induction Machine

Current Harmonic	Synchronous Speed (rpm)
1	1800
3	0
5	$-5 \times 1800 = -9000$
7	$7 \times 1800 = 12{,}600$

The seventh harmonic mmf wave therefore rotates in the same direction as the fundamental wave, but at seven times the speed of the fundamental wave.

Other Harmonic mmf Waves

In general, all odd harmonic mmf waves of order $h = 6m \pm 1$, where m is an integer, are present. These are represented by

$$F_{\mathrm{h}}(\theta, t) = \tfrac{3}{2} F_{h(\max)} \cos(\theta \pm h\omega t) \tag{5.111}$$

The hth-order mmf wave rotates at a speed $h\omega$ radians per second. It rotates in the same direction as the fundamental wave if $h = 6m + 1$, and in the opposite direction if $h = 6m - 1$.

Effects on $T - n$ Characteristic

For a four-pole, three-phase, 60 Hz induction machine the speeds of the rotating mmf waves corresponding to various time harmonic currents are shown in Table 5.1. The torque–speed characteristics corresponding to fundamental, fifth, and seventh harmonic currents are shown in Fig. 5.42. In the normal region of operation of the induction motor, the magnitudes of the parasitic torques are very small. Therefore, time harmonics produce no significant effects on the operation of the induction motor. This is further illustrated in Example 5.10.

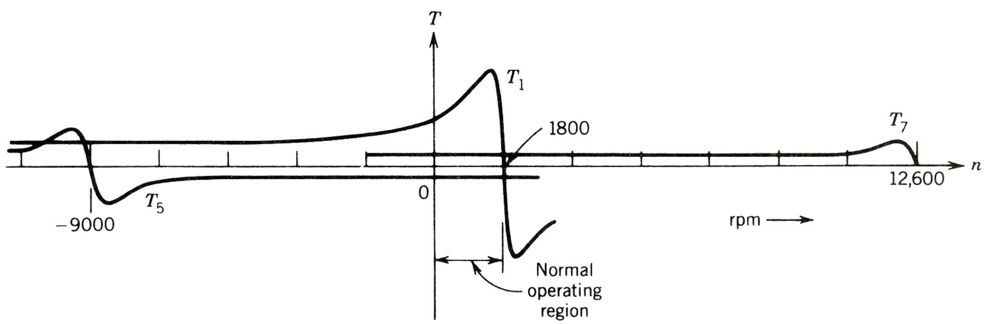


FIGURE 5.42 $T - n$ characteristics for different time harmonic currents.

EXAMPLE 5.11

The speed of a 3ϕ, 5 hp, 208 V, 1740 rpm, 60 Hz, four-pole induction motor is controlled by a current source inverter (Fig. 5.34*a*). The phase current is a quasi-square wave of 120° pulse width, as shown in Fig. 5.34*b*. The phase current can be expressed in a Fourier series as follows:

$$i = 1.1I \sin \omega t - 0.22I \sin 5\omega t - 0.16I \sin 7\omega t + \cdots$$

The parameters of the single-phase equivalent circuit of the induction machine at fundamental frequency (60 Hz) are

$$R_1 = 0.5\ \Omega, \qquad X_1 = X_2' = 1.0\ \Omega$$
$$R_2' = 0.5\ \Omega, \qquad X_{\mathrm{m}} = 35\ \Omega$$

At full load, the induction machine draws a peak current of 10 amps ($=I$ in Fig. 5.34*b*).

(a) Draw the equivalent circuit for the hth harmonic current.

(b) Determine the torques produced by the fundamental current.

(c) Determine the parasitic torques produced by the fifth and seventh harmonic currents.

Solution

(a) The equivalent circuit for the hth harmonic current is shown in Fig. E5.11.

(b) $h = 1$:

$$n_{\mathrm{s}} = \frac{120f}{P} = \frac{120 \times 60}{4} = 1800\ \text{rpm}$$

$$s_1 = \frac{1800 - 1740}{1800} = 0.0333$$

$$\frac{R_2'}{s_1} = \frac{0.5}{0.0333} = 15\ \Omega$$

$$X_{\mathrm{m}} = 35\ \Omega$$

$$X_2' = 1\ \Omega$$

$$Z_1 = \frac{j35(15 + j1)}{15 + j1 + j35}$$
$$= 12.08 + j6.0$$
$$= R_1 + jX_1$$

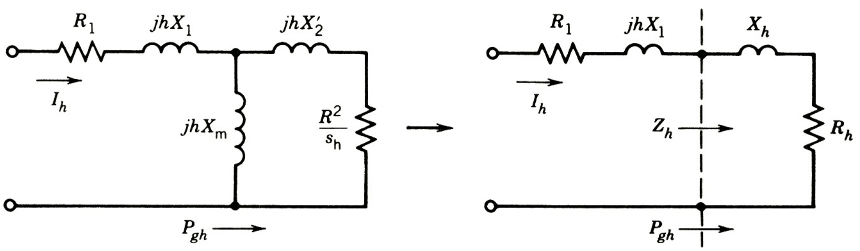


FIGURE E5.11

$$P_{g1} = 3 \times I_1^2 R_1 = 3 \times \left(\frac{1.1 \times 10}{\sqrt{2}}\right)^2 \times 12.08$$

$$= 2192.5\ \mathrm{W}$$

$$T_1 = \frac{P_{g1}}{\omega_{syn}} = \frac{2192.5}{(1800/60) \times 2\pi} = 11.63\ \mathrm{N \cdot m}$$

(c) $h = 5$:

$$n_s = -\frac{120 \times 5 \times 60}{4} = -9000\ \mathrm{rpm}$$

$$s_5 = \frac{-9000 - 1740}{-9000} = 1.19$$

$$\frac{R_2'}{s_5} = \frac{0.5}{1.19} = 0.42\ \Omega$$

$$hX_m = 5 \times 35 = 175\ \Omega$$

$$hX_2' = 5 \times 1 = 5\ \Omega$$

$$Z_5 = \frac{j175(0.42 + j5)}{0.42 + j5 + j175}$$

$$= 0.4 + j4.86\ \Omega$$

$$P_{g5} = 3 \times \left(\frac{0.22 \times 10}{\sqrt{2}}\right)^2 \times 0.4 = 2.9\ \mathrm{W}$$

$$T_5 = \frac{2.9}{-(9000/60) \times 2\pi} = -0.00307\ \mathrm{N \cdot m}$$

$h = 7$:

$$n_s = \frac{120 \times 7 \times 60}{4} = 12{,}600\ \mathrm{rpm}$$

$$s_7 = \frac{12{,}600 - 1740}{12{,}600} = 0.862$$

$$\frac{R_2'}{s_7} = \frac{0.5}{0.862} = 0.58\ \Omega$$

$$hX_m = 7 \times 35 = 245\ \Omega$$

$$hX_2' = 7 \times 1 = 7\ \Omega$$

$$Z_h = \frac{j245(0.58 + j7)}{0.58 + j7 + j245}$$

$$= 0.549 + j6.807\ \Omega$$

$$P_{g7} = 3 \times \left(\frac{0.16 \times 10}{\sqrt{2}}\right)^2 \times 0.549$$

$$= 2.107 \text{ W}$$

$$T_7 = \frac{2.107}{(12{,}600/60) \times 2\pi} = 0.0016 \text{ N} \cdot \text{m}$$

Note that the parasitic torques produced by time harmonics are insignificant compared to the fundamental torque. ■

5.15.2 SPACE HARMONICS

An ideal sinusoidal distribution of mmf is possible only if the machine has an infinitely large number of slots and the turns of a winding are sinusoidally distributed in the slots. This is not practically possible to attain. In a practical machine the winding is distributed in a finite number of slots. As a result, when current flows through a winding, the mmf distribution in space has a stairlike waveform as shown in Fig. 5.43 (see also Appendix A).

The mmf distribution contains a fundamental and a family of space harmonics of order $h = 6m \pm 1$, where m is a positive number. The fundamental and the fifth harmonic component of mmf are also shown in Fig. 5.43.

In a three-phase machine, when sinusoidally varying currents flow through the windings, the space harmonic waves rotate at $(1/h)$ times the speed of the fundamental wave. The space harmonic waves rotate in the same direction as the fundamental wave if $h = 6m + 1$ and in the opposite direction if $h = 6m - 1$.

A space harmonic wave of order h is equivalent to a machine with hp number of poles. Therefore, the synchronous speed of the hth space harmonic wave is

$$n_{s(h)} = \frac{n_s}{h} = \frac{120f}{hp} \tag{5.112}$$

Effects on *T–n* Characteristics

For a three-phase, four-pole, 60 Hz machine, the synchronous speeds of the space harmonics are shown in Table 5.2. The torque–speed characteristics for the fundamental flux and fifth and

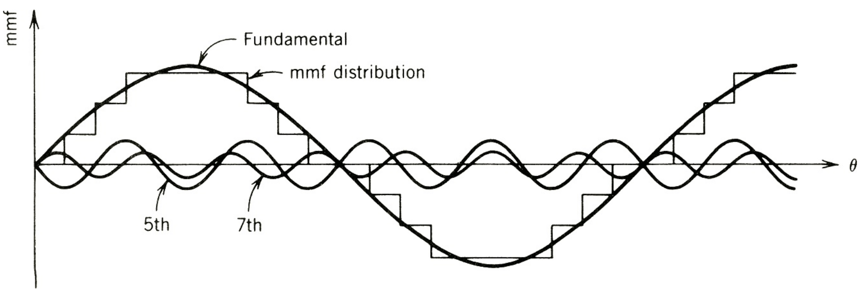


FIGURE 5.43 MMF distribution in the air gap.

TABLE 5.2 Synchronous Speeds for Space Harmonics of a 3ϕ, Four-Pole, 60 Hz Induction Machine

Space Harmonic	Synchronous Speed (rpm)
1	1800
5	$-\frac{1800}{5} = -360$
7	$\frac{1800}{7} = 257.1$
11	$-\frac{1800}{11} = -163.6$
13	$\frac{1800}{13} = 138.5$

seventh space harmonic fluxes are shown in Fig. 5.44. The effects of space harmonics are significant. If the effect of seventh harmonic torque is appreciable, the motor may settle to a lower speed—such as the operating point *A* instead of the desired operating point *B*. The motor therefore crawls. To reduce the crawling effect, the fifth and seventh space harmonics should be reduced, and this can be done by using a chorded (or short-pitched) winding, as discussed in Appendix A.

5.16 LINEAR INDUCTION MOTOR (LIM)

A linear version of the induction machine can produce linear or translational motion. Consider the cross-sectional view of the rotary induction machine shown in Fig. 5.45*a*. Instead of a squirrel-cage rotor, a cylinder of conductor (usually made of aluminum) enclosing the rotor's ferromagnetic core is considered. If the rotary machine of Fig. 5.45*a* is cut along the line *xy* and unrolled, a linear induction machine, shown in Fig. 5.45*b*, is obtained. Instead of the terms

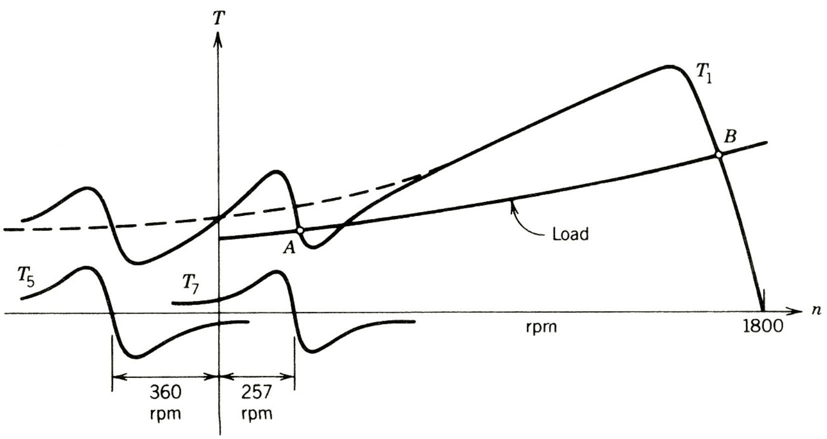


FIGURE 5.44 Parasitic torques due to space harmonics.

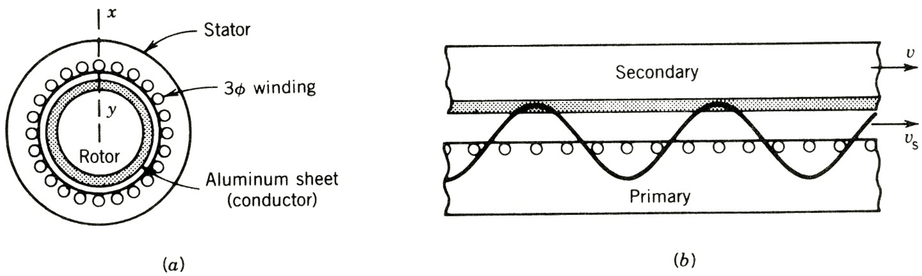


FIGURE 5.45 Induction motors. (*a*) Rotary induction motor. (*b*) Linear induction motor (LIM).

stator and rotor, it is more appropriate to call them primary and secondary members, respectively, of the linear induction machine.

If a three-phase supply is connected to the stator of a rotary induction machine, a flux density wave rotates in the air gap of the machine. Similarly, if a three-phase supply is connected to the primary of a linear induction machine, a traveling flux density wave is created that travels along the length of the primary. This traveling wave will induce current in the secondary conductor. The induced current will interact with the traveling wave to produce a translational force F (or thrust). If one member is fixed and the other is free to move, the force will make the movable member move. For example, if the primary in Fig. 5.45*b* is fixed, the secondary is free to move, and the traveling wave moves from left to right, the secondary will also move to the right, following the traveling wave.

LIM Performance

The synchronous velocity of the traveling wave is

$$V_s = 2T_p f \text{ m/sec} \tag{5.113}$$

where T_p is the pole pitch and f is the frequency of the supply. Note that the synchronous velocity does not depend on the number of poles. If the velocity of the moving member is V, then the slip is

$$s = \frac{V_s - V}{V_s} \tag{5.114}$$

The per-phase equivalent circuit of the linear induction motor has the same form as that of the rotary induction motor as shown in Fig. 5.15. The thrust–velocity characteristic of the linear induction motor also has the same form as the torque–speed characteristic of a rotary induction motor, as shown in Fig. 5.46. The thrust is given by

$$F = \frac{\text{air gap power, } P_g}{\text{synchronous velocity, } V_s}$$

$$= \frac{3I_2'^2 R_2'/s}{V_s} \text{ newtons} \tag{5.115}$$

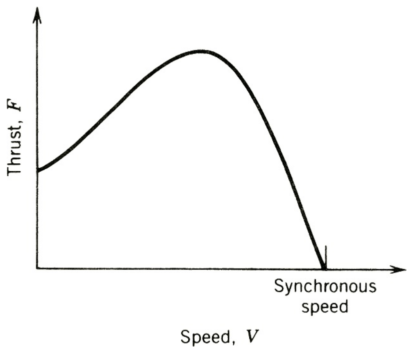


FIGURE 5.46 Thrust–speed characteristic of a LIM.

A linear induction motor requires a large air gap, typically 15–30 mm, whereas the air gap for a rotary induction motor is small, typically 1–1.5 mm. The magnetizing reactance X_m is therefore quite low for the linear induction motor. Consequently, the excitation current is large, and the power factor is low. The LIM also operates at a larger slip. The loss in the secondary is therefore high, making the efficiency low.

The LIM shown in Fig. 5.45*b* is called a *single-sided LIM* or *SLIM*. Another version is used in which primary is on both sides of the secondary, as shown in Fig. 5.47. This is known as a *double-sided LIM* or *DLIM*.

Applications

An important application of a LIM is in transportation. Usually a short primary is on the vehicle and a long secondary is on the track, as shown in Fig. 5.48. A transportation test vehicle using such a LIM is shown in Fig. 5.49.

A LIM can also be used in other applications, such as materials handling, pumping of liquid metal, sliding-door closers, and curtain pullers.

End Effect

Note that the LIM primary has an entry edge at which a new secondary conductor continuously comes under the influence of the magnetic field. The secondary current at the entry edge will

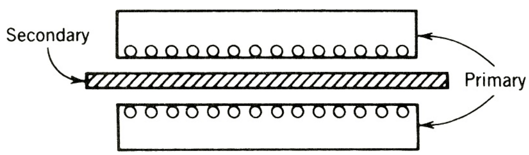


FIGURE 5.47 Double-sided LIM (DLIM).

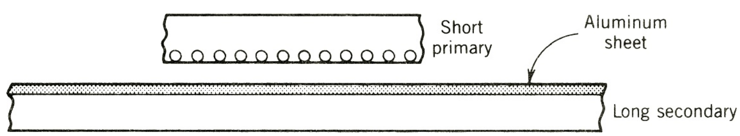
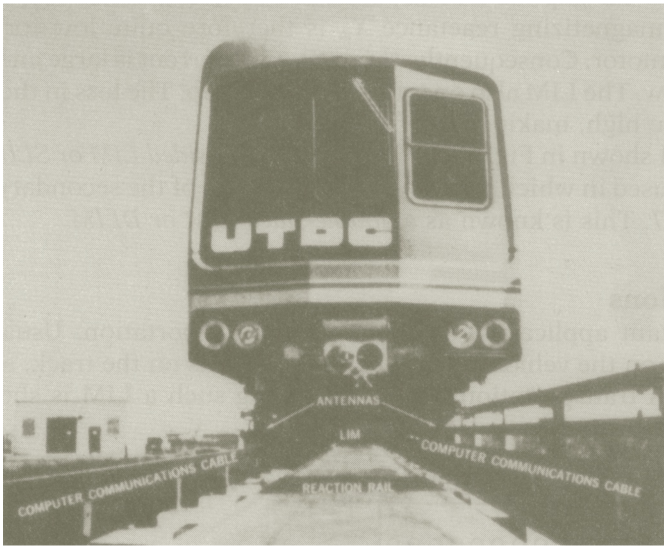


FIGURE 5.48 LIM for a vehicle.



Courtesy of Urban Transportation Development Corporation, Kingston, Canada

FIGURE 5.49 Transportation test vehicle using LIM.

tend to prevent the buildup of air gap flux. As a result, the flux density at the entry edge will be significantly less than the flux density at the center of the LIM. The LIM primary also has an exit edge at which the secondary conductor continuously leaves. A current will persist in the secondary conductor after it has left the exit edge in order to maintain the flux. This current produces extra resistive loss. These phenomena at the entry edge and the exit edge are known as *end effects* in a linear machine. The end effect reduces the maximum thrust that the motor can produce. Naturally, the end effect is more pronounced at high speed.

EXAMPLE 5.12

The linear induction motor shown in Fig. 5.48 has 98 poles and a pole pitch of 50 cm.

(a) Determine the synchronous speed and the vehicle speed in km/hr if frequency is 50 Hz and slip is 0.25.

(b) If the traveling wave moves left to right with respect to the vehicle, determine the direction in which the vehicle will move.

Solution

(a)

$$\begin{aligned} V_s &= 2 \times 50 \times 10^{-2} \times 50 = 50\ \text{m/sec} \\ &= \frac{50 \times 60 \times 60}{1000}\ \text{km/hr} \\ &= 180\ \text{km/hr} \\ V &= (1 - 0.25)180 = 135\ \text{km/hr} \end{aligned}$$

(b) Right to left. ■

PROBLEMS

5.1 A 3ϕ, 460 V, 60 Hz, 4-pole, Y-connected wound-rotor induction machine has 230 V between the slip rings at standstill when the stator is connected to a 3ϕ, 460 V, 60 Hz power supply. The rotor is mechanically connected to a dc motor whose speed can be changed. Determine the magnitude and frequency of the voltage between the slip rings when the induction machine is driven at the following speeds:

(a) 1620 rpm in the same direction as the rotating field.

(b) 1620 rpm in the opposite direction to the rotating field.

(c) 1800 rpm in the same direction as the rotating field.

(d) 1800 rpm in the opposite direction to the rotating field.

(e) 3600 rpm in the same direction as the rotating field.

5.2 A three-phase, 5 hp, 208 V, 60 Hz induction motor runs at 1746 rpm when it delivers rated output power.

(a) Determine the number of poles of the machine.

(b) Determine the slip at full load.

(c) Determine the frequency of the rotor current.

(d) Determine the speed of the rotor field with respect to the

(i) Stator.

(ii) Stator rotating field.

5.3 A 3ϕ, 460 V, 100 hp, 60 Hz, six-pole induction machine operates at 3% slip (positive) at full load.

(a) Determine the speeds of the motor and its direction relative to the rotating field.

(b) Determine the rotor frequency.

(c) Determine the speed of the stator field.

(d) Determine the speed of the air gap field.

(e) Determine the speed of the rotor field relative to

(i) the rotor structure.

(ii) the stator structure.

(iii) the stator rotating field.

5.4 Repeat Problem 5.3 if the induction machine is operated at 3% slip (negative).

5.5 Repeat Problem 5.3 if the induction machine operates at 150% slip (positive).

5.6 A 3ϕ, 10 hp, 208 V, six-pole, 60 Hz, wound-rotor induction machine has a stator-to-rotor turns ratio of 1 : 0.5 and both stator and rotor windings are connected in star.

(a) The stator of the induction machine is connected to a 3ϕ, 208 V, 60 Hz supply, and the motor runs at 1140 rpm.

(i) Determine the operating slip.

(ii) Determine the voltage induced in the rotor per phase and frequency of the induced voltage.

(iii) Determine the rpm of the rotor field with respect to the rotor and with respect to the stator.

(b) If the stator terminals are shorted and the rotor terminals are connected to a 3ϕ, 208 V, 60 Hz supply and the motor runs at 1164 rpm,

  (i) Determine the direction of rotation of the motor with respect to that of the rotating field.

  (ii) Determine the voltage induced in the stator per phase and its frequency.

5.7 A 3ϕ, 460 V, 60 Hz, 20 kW induction machine draws 25 A at a power factor of 0.9 lagging when connected to a 3ϕ, 460 V, 60 Hz power supply. The core loss is 900 W, stator copper loss is 1100 W, rotor copper loss is 550 W, and friction and winding loss is 300 W. Calculate

(a) The air gap power, P_{ag}.

(b) The mechanical power developed, P_{mech}.

(c) The output horse power.

(d) The efficiency.

5.8 The machine in Problem 5.7 is a 4-pole machine. For the operating condition of Problem 5.7, determine

(a) The synchronous speed, n_{s}.

(b) The rotor speed, n.

(c) The developed torque.

(d) The output torque.

5.9 A 3ϕ, 2-pole, 60 Hz, induction motor operates at 3546 rpm while delivering 20 kW to a load. Neglect all losses. Determine

(a) The slip of the motor.

(b) The developed torque.

(c) The speed of the motor if the torque is doubled. Assume that in the low slip region, the torque speed curve is linear.

(d) The power supplied by the motor for the load condition of (c).

5.10 A 3ϕ, 6-pole, 60 Hz, induction motor runs at 1140 rpm and draws 90 kW from the supply. The core loss and copper losses in the stator is 4.8 kW. The windage and friction loss is 2.1 kW. Calculate

(a) The air gap power.

(b) The rotor copper loss.

(c) The mechanical power delivered to the load.

(d) The load torque.

(e) The efficiency.

5.11 A 3ϕ, 10 hp, 460 V, 60 Hz, 4-pole induction motor runs at 1730 rpm at full-load. The stator copper loss is 200 W and the windage and friction loss is 320 W. Determine

(a) The mechanical power developed, P_{mech}.

(b) The air gap power, P_{ag}.

(c) The rotor copper loss, P_{cu2}.

(d) The input power, P_{in}.

(e) The efficiency of the motor.

5.12 A 3ϕ, 2.5 hp, 210 V, 60 Hz, 4-pole induction motor runs at 1700 rpm. The rotational losses are 150 W. If the rotor resistance per phase is 0.025 Ω, determine the rotor current.

5.13 The following test results are obtained from a 3ϕ, 100 hp, 460 V, eight-pole, star-connected squirrel-cage induction machine.

No-load test:	460 V, 60 Hz, 40 A, 4.2 kW
Blocked-rotor test:	100 V, 60 Hz, 140 A, 8.0 kW

Average dc resistance between two stator terminals is 0.152 Ω.

(a) Determine the parameters of the equivalent circuit.

(b) The motor is connected to a 3ϕ, 460 V, 60 Hz supply and runs at 873 rpm. Determine the input current, input power, air gap power, rotor copper loss, mechanical power developed, output power, and efficiency of the motor.

5.14 The following test results are obtained for a 3ϕ, 280 V, 60 Hz, 6.5 A, 500 W induction machine.

Blocked-rotor test:	44 V, 60 Hz, 25 A, 1250 W
No-load test:	208 V, 60 Hz, 6.5 A, 500 W

The average resistance measured by a dc bridge between two stator terminals is 0.54 Ω.

(a) Determine the no-load rotational loss.

(b) Determine the parameters of the equivalent circuit.

(c) What type of induction motor is this?

(d) Determine the output horsepower at $s = 0.1$.

5.15 A 3ϕ, 280 V, 60 Hz, 20 hp, four-pole induction motor has the following equivalent circuit parameters.

$$R_1 = 0.12\,\Omega \qquad R_2' = 0.1\,\Omega$$
$$X_1 = X_2' = 0.25\,\Omega$$
$$X_m = 10.0\,\Omega$$

The rotational loss is 400 W. For 5% slip, determine

(a) The motor speed in rpm and radians per sec.

(b) The motor current.

(c) The stator cu-loss.

(d) The air gap power.

(e) The rotor cu-loss.

(f) The shaft power.

(g) The developed torque and the shaft torque.

(h) The efficiency.

Use the IEEE-recommended equivalent circuit (Fig. 5.15).

5.16 The motor in Example 5.5 is taken to Europe where the supply frequency is 50 Hz.

(a) What supply voltage is to be used, and why?

(b) The motor is operated with the supply voltage of part (a) and at a slip of 3%.

(i) Determine synchronous speed, motor speed, and rotor frequency.

(ii) Determine motor current, power factor, torque developed, and efficiency. Assume rotational loss to be proportional to motor speed.

5.17 A 3ϕ, 460 V, 60 Hz, six-pole induction motor has the following single-phase equivalent circuit parameters:

$$R_1 = 0.2\,\Omega \qquad X_1 = 1.055\,\Omega$$
$$R_2' = 0.28\,\Omega \qquad X_2' = 1.055\,\Omega$$
$$X_m = 33.9\,\Omega$$

The induction motor is connected to a 3ϕ, 460 V, 60 Hz supply.

(a) Determine the starting torque.

(b) Determine the breakdown torque and the speed at which it occurs.

(c) The motor drives a load for which $T_L = 1.8\,\text{N}\cdot\text{m}$. Determine the speed at which the motor will drive the load. Assume that near the synchronous speed the motor torque is proportional to slip. Neglect rotational losses.

Use the approximate equivalent circuit of Fig. 5.14*b*.

5.18 A 3ϕ, 208 V, 60 Hz, six-pole induction motor has the following equivalent circuit parameters:

$$R_1 = 0.075\,\Omega \qquad R_2' = 0.11\,\Omega$$
$$L_1 = L_2' = 0.25\,\text{mH}$$
$$L_m = 15.0\,\text{mH}$$

The motor drives a fan. The torque required for the fan varies as the square of the speed and is given by

$$T_{\text{fan}} = 12.7 \times 10^{-3}\omega_m^2$$

Determine the speed, torque, and power of the fan when the motor is connected to a 3ϕ, 208 V, 60 Hz supply. Use the approximate equivalent circuit of Fig. 5.14*b*, and neglect rotational losses. For operation at low slip, the motor torque can be considered proportional to slip.

5.19 A 3ϕ, 100 kVA, 460 V, 60 Hz, eight-pole induction machine has the following equivalent circuit parameters:

$$R_1 = 0.07\,\Omega, \qquad X_1 = 0.2\,\Omega$$
$$R_2' = 0.05\,\Omega, \qquad X_2' = 0.2\,\Omega$$
$$X_m = 6.5\,\Omega$$

(a) Derive the Thevenin equivalent circuit for the induction machine.

(b) If the machine is connected to a 3ϕ, 460 V, 60 Hz supply, determine the starting torque, the maximum torque the machine can develop, and the speed at which the maximum torque is developed.

(c) If the maximum torque is to occur at start, determine the external resistance required in each rotor phase. Assume a turns ratio (stator to rotor) of 1.2.

5.20 A 3ϕ, 25 hp, 460 V, 60 Hz, 1760 rpm, wound-rotor induction motor has the following equivalent circuit parameters:

$$R_1 = 0.25\,\Omega, \qquad X_1 = 1.2\,\Omega$$
$$R_2' = 0.2\,\Omega, \qquad X_2' = 1.1\,\Omega$$
$$X_{\mathrm{m}} = 35.0\,\Omega$$

The motor is connected to a 3ϕ, 460 V, 60 Hz, supply.

(a) Determine the number of poles of the machine.

(b) Determine the starting torque.

(c) Determine the value of the external resistance required in each phase of the rotor circuit such that the maximum torque occurs at starting. Use Thevenin's equivalent circuit.

5.21 Repeat Problem 5.20 using the approximate equivalent circuit of Fig. 5.14*b*.

5.22 A three-phase, 460 V, 60 Hz induction machine produces 100 hp at the shaft at 1746 rpm. Determine the efficiency of the motor if rotational losses are 3500 W and stator copper losses are 3000 W.

5.23 A 440 V, 60 Hz, six-pole, 3ϕ induction motor is taking 50 kVA at 0.8 power factor and is running at a slip of 2.5%. The stator copper losses are 0.5 kW and rotational losses are 2.5 kW. Compute

(a) The rotor copper losses.

(b) The shaft hp.

(c) The efficiency.

(d) The shaft torque.

5.24 A 3ϕ wound-rotor induction machine is mechanically coupled to a 3ϕ synchronous machine as shown in Fig. P5.24. The synchronous machine has four poles, and the induction machine has six poles. The stators of the two machines are connected to a 3ϕ, 60 Hz power supply. The rotor of the induction machine is connected to a 3ϕ resistive load. Neglect rotational losses and stator resistance losses. The load power is 1 pu. The synchronous machine rotates at the synchronous speed.

(a) The rotor rotates in the direction of the stator rotating field of the induction machine. Determine the speed, frequency of the current in the resistive load, and power taken by the synchronous machine and by the induction machine from the source.

(b) Repeat (a) if the phase sequence of the stator of the induction machine is reversed.

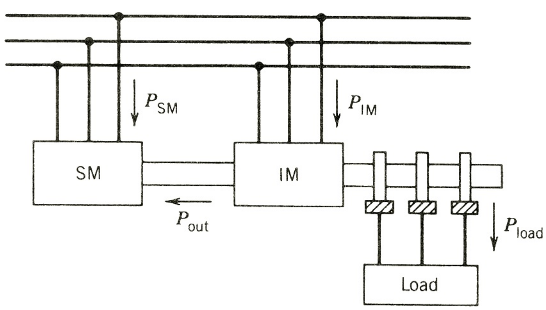


FIGURE P5.24

5.25 A 3ϕ induction machine is mechanically coupled to a dc shunt machine. The rating and parameters of the machines are as follows:

Induction machine:

3ϕ, 5 kVA, 208 V, 60 Hz, four-pole, 1746 rpm

$R_1 = 0.25\,\Omega$, $X_1 = 0.55\,\Omega$, $R_2' = 0.35\,\Omega$, $X_2' = 1.1\,\Omega$, $X_m = 38\,\Omega$

DC machine:

220 V, 5 kW, 1750 rpm

$R_a = 0.4\,\Omega$, $R_{fw} = 100\,\Omega$, $R_{fc} = 100\,\Omega$

The induction machine is connected to a 3ϕ, 208 V, 60 Hz supply, and the dc machine is connected to a 220 V dc supply. The rotational loss of each machine of the M–G set may be considered constant at 225 W.

The system rotates at 1710 rpm in the direction of the rotating field of the induction machine.

(a) Determine the mode of operation of the induction machine.

(b) Determine the current taken by the induction machine.

(c) Determine the real and reactive power at the terminals of the induction machine and indicate their directions.

(d) Determine the copper loss in the rotor circuit.

(e) Determine the armature current (and its direction) of the dc machine.

5.26 The field current of the dc machine in the M–G set of Problem 5.25 is decreased so that the speed of the set increases to 1890 rpm. Repeat parts (a) to (e) of Problem 5.25.

5.27 The M–G set in Problem 5.25 is rotating at 1710 rpm in the direction of the rotating field. The phase sequence of the supply connected to the induction machine is suddenly reversed. Repeat parts (a) to (e) of Problem 5.25.

5.28 A 3ϕ, 250 kW, 460 V, 60 Hz, eight-pole induction machine is driven by a wind turbine. The induction machine has the following parameters.

$$R_1 = 0.015\,\Omega, \qquad R_2' = 0.035\,\Omega$$
$$L_1 = 0.385\text{ mH}, \qquad L_2' = 0.358\text{ mH}, \qquad L_m = 17.24\text{ mH}$$

The induction machine is connected to a 460 V infinite bus through a feeder having a resistance of $0.01\,\Omega$ and an inductance of 0.0 8 mH. The wind turbine drives the induction machine at a slip of -25%.

(a) Determine the speed of the wind turbine.

(b) Determine the voltage at the terminals of the induction machine.

(c) Determine the power delivered to the infinite bus and the power factor.

(d) Determine the efficiency of the system. Assume the rotational and core losses to be 3 kW.

5.29 The motor of Example 5.5 is running at rated (full-load) condition. The motor is stopped by plugging (for rapid stopping)—that is, switching any two stator leads and removing the power from the motor at the moment the rotor speed goes through zero. Determine the following, at the time immediately after switching the stator leads:

(a) The slip.

(b) The rotor circuit frequency.

(c) The torque developed and its direction with respect to rotor motion.

5.30 A 3ϕ, 460 V, 250 hp, eight-pole wound-rotor induction motor controls the speed of a fan. The torque required for the fan varies as the square of the speed. At full load (250 hp) the motor slip is 0.03 with the slip rings short-circuited. The slip–torque relationship of the motor can be assumed to be linear from no load to full load. The resistance of each rotor phase is 0.02 ohms. Determine the value of resistance to be added to each rotor phase so that the fan runs at 600 rpm.

5.31 A 3ϕ, squirrel-cage induction motor has a starting torque of 1.75 pu and a maximum torque of 2.5 pu when operated from rated voltage and frequency. The full-load torque is considered as 1 pu of torque. Neglect stator resistance.

(a) Determine the slip at maximum torque.

(b) Determine the slip at full-load torque.

(c) Determine the rotor current at starting in per unit—consider the full-load rotor current as 1 pu.

(d) Determine the rotor current at maximum torque in per unit of full-load rotor current.

5.32 A 3ϕ, 460 V, 60 Hz, four-pole wound-rotor induction motor develops full-load torque at a slip of 0.04 when the slip rings are short-circuited. The maximum torque it can develop is 2.5 pu. The stator leakage impedance is negligible. The rotor resistance measured between two slip rings is 0.5 Ω.

(a) Determine the speed of the motor at maximum torque.

(b) Determine the starting torque in per unit. (Full-load torque is one per-unit torque.)

(c) Determine the value of resistance to be added to each phase of the rotor circuit so that maximum torque is developed at the starting condition.

(d) Determine the speed at full-load torque with the added rotor resistance of part (c).

5.33 The approximate per-phase equivalent circuit for a 3ϕ, 60 Hz, 1710 rpm double-cage rotor induction machine is shown in Fig. P5.33. The standstill rotor impedances referred to the stator are as follows:

Outer cage:	$4.0 + j1.5\,\Omega$
Inner cage:	$0.5 + j4.5\,\Omega$

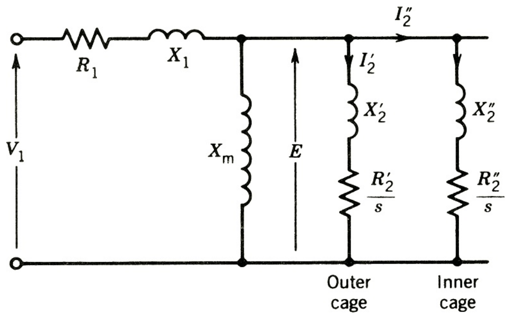


FIGURE P5.33

If stator impedance is neglected,

(a) Determine the ratio of currents in the outer and inner cages for standstill and full-load conditions.

(b) Determine the starting torque of the motor as percent of the full-load torque.

(c) Determine the ratio of torques due to the outer and inner cages for standstill and full-load conditions.

5.34 A 3ϕ, 460 V, 60 Hz, four-pole double-cage induction motor has the following equivalent circuit parameters (Fig. 5.27*b*):

$$L_m = 134\ \text{mH} \qquad R_2' = 4.8\ \Omega$$
$$L_2' = 4.51\ \text{mH}, \qquad R''_2 = 1.2\ \Omega$$
$$L_2'' = 11.41\ \text{mH}$$

The stator impedance (R_1 and X_1) may be neglected. The motor rotates at 95% of synchronous speed. Determine the torques produced by the inner cage and the outer cage and the total torque.

5.35 The motor in Example 5.5 is connected in Y. If the motor is connected in Δ, what would be the starting current?

5.36 A 3ϕ, 200 hp, 460 V, 1760 rpm, 60 Hz induction motor has a power factor of 0.85 lagging and an efficiency of 90% at full load. If started with rated voltage, the starting current is six times larger than the rated current of the motor. An autotransformer is used to start the motor at reduced voltage.

(a) Determine the rated motor current.

(b) Determine the autotransformer output voltage to make the motor starting current twice the full-load current.

(c) Determine the ratio of the starting torque at the reduced voltage of part (b) to the torque at rated voltage.

5.37 A 3ϕ, 460 V, 60 Hz, 1755 rpm, 100 hp, four-pole squirrel-cage induction motor has negligible stator resistance and leakage inductance. The motor is to be operated from a 50 Hz supply.

(a) Determine the supply voltage if the air gap flux is to remain at the same value if it were operated from a 3ϕ, 460 V, 60 Hz supply.

(b) Determine the speed at full-load torque if the motor operates from the 50 Hz supply of part (a).

5.38 A 3ϕ, 460 V, 60 Hz, 50 hp, 1180 rpm induction motor has the following parameters:

$R_1 = 0.191\ \Omega$

$R_2' = 0.0707\ \Omega$

$L_1 = 2\ \text{mH}$ (stator leakage inductance)

$L_2' = 2\ \text{mH}$ (rotor leakage inductance, referred to stator)

$L_m = 44.8\ \text{mH}$

(a) Determine the values of the rated current and rated torque (use the equivalent circuit of Fig. 5.15).

(b) Use

$$I_{\text{rated}} = 1 \text{ pu of current}$$
$$T_{\text{rated}} = 1 \text{ pu of torque}$$
$$V_{\text{rated}} = 1 \text{ pu of voltage}$$
$$n_{\text{syn}} = 1 \text{ pu of speed}$$

Plot in per-unit values torque versus speed for $V_1 = 1$ pu and stator frequency $f_1 = 60$ Hz. On the same graph, plot in per-unit values torque versus speed for $I_1 = 1$ pu and $f_1 = 60$ Hz.

5.39 For the induction machine of Example 5.5,

(a) Determine the maximum torques (in newton · meters as well as in per unit) developed by the induction motor for the following operations.

(i) Motor operated from a 3ϕ, 460 V, 60 Hz supply.

(ii) Motor operated under constant-flux operation. Consider E/f for the full-load operating condition of Example 5.4.

(iii) Motor operated under constant current of rated value.

(b) Write a computer program and plot the torque versus f_2 for the three conditions of part (a). Vary f_2 from zero to 60 Hz.

5.40 The speed of a 3ϕ, 5 hp, 208 V, 60 Hz, four-pole induction motor is controlled by a voltage source inverter. The phase voltage has the following component voltages:

Fundamental voltage, rms = 100 V.

Fifth harmonic voltage, rms = 15 V.

Seventh harmonic voltage, rms = 10 V.

The parameters of the single-phase equivalent circuit of the induction machine at fundamental frequency (60 Hz) are as follow:

$$R_1 = \text{negligible}, \qquad R_2' = 0.5\,\Omega$$
$$X_1 = \text{negligible}, \qquad X_2' = 1.0\,\Omega$$
$$X_m = 35\,\Omega$$

The induction motor is loaded, and it rotates at 1710 rpm.

(a) Determine the torque produced by the fundamental voltage.

(b) Determine the torque produced by the fifth harmonic voltage.

(c) Determine the torque produced by the seventh harmonic voltage.

5.41 The speed of a 3ϕ, 5 hp, 208 V, 60 Hz, four-pole induction motor is controlled by a voltage source inverter. The phase voltage has the following component voltages:

Fundamental voltage, rms = 100 V.

Fifth harmonic voltage, rms = 18 V.

Seventh harmonic voltage, rms = 12 V.

The parameters of the single-phase equivalent circuit of the induction machine at fundamental frequency (60 Hz) are

$$R_1 = 0.7\,\Omega, \qquad R_2' = 0.6\,\Omega$$
$$X_1 = X_2' = 1.2\,\Omega$$
$$X_m = \text{very large}$$

The induction motor is loaded, and it rotates at 1710 rpm.

(a) Determine the torque produced by the fundamental voltage.

(b) Determine the torque produced by the fifth harmonic voltage.

5.42 **(a)** What causes space harmonics in an ac machine? Briefly describe their effects on the torque–speed characteristic of a 3ϕ induction motor.

(b) Determine the synchronous speed (in rpm) for the fifth space harmonic flux wave of a 3ϕ, eight-pole, 60 Hz induction motor. What is its direction of motion with respect to the fundamental space flux wave?

5.43 The linear induction motor shown in Fig. 5.48 has 60 poles and has a pole pitch of 50 cm.

(a) Determine the synchronous speed and the vehicle speed in km/hr if the frequency is 50 Hz and the slip is 0.25.

(b) If the traveling wave moves left to right with respect to the vehicle, determine the direction in which the vehicle will move.

5.44 A LIM has seven poles, and the pole pitch is 30 cm. The parameters of the single-phase equivalent circuit are

$$R_1 = 0.15\,\Omega, \qquad R_2' = 0.25\,\Omega$$
$$L_1 = 0.5\,\text{mH}, \qquad L_2' = 0.8\,\text{mH}$$
$$L_m = 5.0\,\text{mH}$$

The LIM is connected to a 3ϕ variable-voltage, variable-frequency supply. At 300 V, 50 Hz, the speed of the LIM is 75 km/hr.

(a) Determine the slip.

(b) Determine the input current, input power, power factor, air gap power, mechanical power developed, power loss in the secondary, and thrust produced.

5.45 A LIM has ten poles, and the pole pitch is 30 cm. The parameters of the single-phase equivalent circuit are

$$R_1 = 0.15\,\Omega, \qquad R_2' = 0.25\,\Omega$$
$$L_1 = 0.5\,\text{mH}, \qquad L_2' = 0.8\,\text{mH}$$

The LIM is fed from a current source inverter, and it drives a vehicle. The controller, shown in Fig. 5.35, keeps the rotor frequency constant at $f_2 = 5$ Hz. The fundamental LIM current is 200 A (rms). Neglect the effects of the harmonic currents on thrust. For $f_1 = 60$ Hz and $f_2 = -5$ Hz, determine

(a) The mode of operation—that is, motoring or generating.

(b) The value of f_n.

- **(c)** The slip.
- **(d)** The cruising speed (velocity) in km/hr.
- **(e)** The air gap power and its direction of flow.
- **(f)** The thrust.

5.46 Repeat Problem 5.45 for $f_2 = +5$ Hz.

5.47 For Problem 5.46, determine the starting thrust.

5.48 A 3ϕ, 460 V, 60 Hz, 1025 rpm squirrel-cage induction motor has the following equivalent circuit parameters:

$$R_1 = 0.06\,\Omega, \qquad X_1 = 0.25\,\Omega$$
$$R_2' = 0.3\,\Omega, \qquad X_2' = 0.35\,\Omega$$
$$X_m = 7.8\,\Omega$$

Neglect the core losses and windage and friction losses. Use the equivalent circuit of Fig. 5.15 for computation. Write a computer program to study the performance characteristics of this machine operating as a motor over the speed range zero to synchronous speed. The program should yield

- **(a)** A computer printout in tabular form showing the variation of torque, input current, input power factor, and efficiency with speed.
- **(b)** A plot of the performance characteristics.
- **(c)** Input current, torque, input power factor, and efficiency at the rated speed of 1025 rpm.